Robust coordinated transmission for cooperative small cell networks

Yonggang Kim\textsuperscript{1}, Kyung-Joon Park\textsuperscript{2}, Daeyoung Park\textsuperscript{3}, Hyuk Lim\textsuperscript{1} \footnote{E-mail: hlim@gist.ac.kr}

\textsuperscript{1}School of Electrical Engineering and Computer Science, Gwangju Institute of Science and Technology (GIST), Gwangju 500-712, Republic of Korea
\textsuperscript{2}Department of Information and Communication Engineering, Daegu Gyeongbuk Institute of Science and Technology (DGIST), Daegu 711-873, Republic of Korea
\textsuperscript{3}Department of Information and Communication Engineering, Inha University, Incheon 402-751, Republic of Korea

Abstract: Within a macrocell with a large coverage area, multiple small cells are deployed such that each small cell base station (SBS) supports wireless service demands from user equipments (UEs). Each UE can be simultaneously served by multiple SBSs with quality of service (QoS) enhancement. When there exist hotspot areas with a number of UEs, the SBSs near the hotspot areas may experience a higher resource utilisation level than those outside of the hotspot areas, resulting in a shortage of available resources. The authors propose a robust resource-utilisation-based coordinated transmission for heterogeneous networks with a locally different level of traffic demands. In the utilisation-based coordinated transmission, low-utilisation SBSs with a small number of UEs are selected to serve a newly joining UE because they have more capacity to serve requests with bursty traffic demand. They further formulate the selection of cooperative SBSs as a robust optimisation problem in order to ensure that UEs have sufficiently high signal-to-interference-plus-noise ratios, even with channel estimation inaccuracy and strong interference from non-cooperative SBSs. The simulation results indicate that the proposed method guarantees robust and efficient service performance in heterogeneous small cell networks.

1 Introduction

In a macrocell with a large coverage area, it will be difficult for a single macrocell base station (MBS) to satisfy high traffic demand and quality of service (QoS) requirements for user equipments (UEs) when the MBS becomes overloaded with excessive service requests from UEs. In order to prevent a single MBS from being overwhelmed with service requests, multiple base stations (BSs) with low transmit power can be deployed within a macrocell [1, 2]. In general, this heterogeneous network consists of a single macrocell and multiple small cells, and the BS in each small cell is intended to provide wireless service in certain small areas with high traffic demand, such as a hotspot area. Since the traffic load is distributed over multiple BSs, a heterogeneous network can achieve better network performance than a single macrocell. However, because each small cell BS (SBS) usually uses a low transmit power to avoid interfering with UEs associated with an MBS, the UEs served by the single SBS may not have signal-to-interference-plus-noise ratios (SINRs) high enough for successful data transmission. In this case, a UE can be simultaneously served by multiple SBSs, and the SBS can cooperatively transmit a signal to the UE [3–13].

In coordinated transmission schemes, higher diversity in BS coordination can be achieved as the number of cooperative SBSs increases, and thus the throughput performance can be significantly improved. For coordinated transmission, the cooperative SBSs exchange channel state information (CSI) or transmit messages through a wired backbone. However, the increased number of cooperative BSs introduces information exchange overhead among the BSs [3–6]. Therefore, it is critical to select an appropriate subset of BSs to participate in coordinated transmission, rather than all the BSs in a network.

In this paper, we consider a heterogeneous coordinated transmission scenario, where there exist hotspot areas with high demand for wireless services for a large number of UEs while the number of UEs rapidly decreases at the outside of hotspot areas. In fact, heterogeneous networks with hotspot areas are actively being studied, because it is challenging to satisfy the QoS requirements for a large number of UEs in hotspot areas [14, 15]. In [14], it was shown that heavy traffic demands in hotspot areas may degrade the network performance, and in order to successfully take care of excessive service requests from UEs, SBSs should be appropriately deployed in locations where the interference from an MBS is not strong. In [15], it was proposed to exploit mobile SBSs in heterogeneous networks in order to satisfy the QoS requirements for UEs in hotspot areas. However, because SBSs deployed in hotspot areas are easily overloaded with a large number of UEs, the QoS requirement for UEs may not be satisfied in the hotspot areas. To avoid service performance degradation, we propose to use the level of SBS resource utilisation as a cooperative SBS selection metric. The resource utilisation of an SBS is given by the ratio of the number of currently occupied subcarriers to the total number of subcarriers. As a BS provides wireless service to more UEs, the resource utilisation of the BS increases. If the utilisation of a BS is close to 1, the QoS for UEs in its service area declines, especially when the traffic demands from UEs are bursty. If there are nearby SBSs with a small number of UEs, it would be better to include them in the set of cooperative SBSs even if they are not the closest ones to the UEs. This utilisation-based approach also leads to traffic load balancing among SBSs in a hotspot scenario.

In a coordinated transmission, when interference from non-cooperative BSs – e.g. BSs in a macrocell – is strong, a UE may not have a sufficient SINR for successful communication. Even when a set of SBSs is selected to guarantee a sufficient SINR at a UE, the SINR could fall to unacceptable levels if the channel estimation is inaccurate. Therefore it is of critical importance to compensate for inaccurate channel estimation and interference to ensure robust service performance. To this end, we apply robust optimisation to the coordinated transmission that can give a robust solution even when there exists uncertainty in parameter estimation. Simulation results indicate that this robust
approach outperforms the conventional methods that use a static margin for SINR estimation.

The remainder of this paper is organised as follows. In Section 2, we provide an overview of related work. In Section 3, we present the system model and derive the SINR for each UE when multiple SBSs cooperatively provide service to the same UE. Then, in Section 4, we explain the proposed cooperative transmission method. We also derive the upper bound of outage probability. In Section 5, we present our performance evaluation, and our conclusion follows in Section 6.

2 Related work

There are a large number of actively ongoing studies on coordinated transmission among BSs for improving network service performance.

2.1 Coordinated transmission approach

2.1.1 Static topology-based coordinated transmission: The coordinated transmission among cooperating BSs increases the system complexity owing to the information exchange. To reduce the complexity and to exploit the benefits of the coordinated transmission, Marsch and Fettweis [3] focused on static topology-based coordinated transmission. They formulated an average SINR maximisation problem according to a predefined cluster, and presented that the appropriate static clustering for a given network topology shows a network throughput performance close to the user-centric clustering while requiring low control overhead. Huang and Andrews [4] studied the SINR outage probability of UEs when the UEs are serviced by static clustered BSs. They assumed a Poisson distributed BS topology and clustered BSs as a hexagonal lattice. The analytic results showed that the outage probability largely depends on the average number of cooperating BSs in each cluster, and presented that the number of cooperating BSs in each cluster is important in static clustering. Katranaras et al. [7] studied a static topology-based coordinated transmission in long-term evolution (LTE) networks. They analysed the power consumption for the transmission, signal processing and backhaul link connection of cooperating BSs in a variety of static cluster sizes and deployment densities of cooperating BSs. Through the simulation results, they showed that an appropriate predefined cluster size of BSs can improve the energy efficiency for a given BS density of a network.

2.1.2 User-centric clustering-based coordinated transmission: To enhance the throughput performance of the UEs, extensive research has been carried out on how to select a proper set of BSs for each UE while maintaining a certain SINR at the UEs. Zhao et al. [8] studied an overlapped coordinated transmission in LTE-Advanced to order to determine an effective set of cooperating BSs for each cell-edge user. The proposed algorithm in [8] computes the SINR gain of each cell-edge user for every possible cluster, and selects the best cluster that gives the highest SINR gain to the user. Through simulation results, they showed that user-centric clustering-based coordinated transmission achieves better network throughput performance than static topology-based coordinated transmission. Baracca et al. [9] considered the problem of dynamic joint clustering and scheduling for BSs and UEs for downlink coordinated transmission. Based on SINR values, UEs are grouped according to the preferred BS. On the basis of the preferred BS set for UEs, a greedy clustering selection algorithm that iteratively determines BS clusters at each time slot was proposed to improve service performance by mitigating the interference among BS clusters. In [10], Garcia et al. analysed SINR outage probability caused by signals from non-cooperative BSs and proposed a clustering algorithm that maximises the normalised goodput. They presented that the proposed user-centric clustering algorithm achieves higher normalised goodput than static topology-based clustering.

2.2 Coordinated transmission strategy

2.2.1 Energy consumption minimisation: There has been a line of research on energy consumption reduction in BS coordinated transmission. Huang and Ansari studied the influence of the number of UEs served by multiple BSs on network performance, and proved that the proper number of UEs associated with multiple BSs increases energy efficiency [11]. They formulated a power consumption minimisation problem and proposed a joint spectrum and power allocation (JSPA) algorithm that selects the cooperating BSs while restricting the number of UEs associated with multiple BSs. Han et al. [12] investigated the benefits of coordinated transmission in multicell cooperative networks where underutilised BSs can go into a sleep mode to reduce energy consumption. They proposed a power and subcarrier allocation algorithm that minimises network power consumption, retaining sufficient SINR for the wireless service. He et al. [13] studied distributed energy-efficient coordinated transmission. They defined energy efficiency as the ratio of the transmission rate sum to the total power consumption, and formulated an energy efficiency maximisation problem as a fractional programming problem. They decomposed the problem into a master problem and subproblems for each BS and proposed a power allocation algorithm for solving the decomposed problem.

2.2.2 Network overhead minimisation: The amount of information exchange among BSs has been one of the key research issues because network overhead may increase severely with the number of cooperating BSs. In [5], Unachukwu et al. investigated the impact of the number of cooperating BSs per UE on power consumption and data overhead. They showed how the number of cooperating BSs per UE influences network performance, and presented that a proper restriction on cooperating BS set size improves energy efficiency. In [6], Zhao et al. focused on the problem of minimising user data transfer in the backhaul under QoS and BS power constraints. They defined a routing matrix for distributing user data to cooperating BSs and formulated a routing matrix minimisation problem as an $l_0$-norm minimisation problem. Due to $l_0$-norm minimisation is NP-hard, the authors proposed two algorithms based on $l_1$-norm minimisation and $l_2$-norm relaxation, and they showed that the algorithms can significantly reduce user data transfer in the backhaul.

2.3 Our contribution

Our main contributions are summarised as follows:

• We consider resource utilisation as a new performance metric for user scheduling in the optimisation of coordinate transmission. In previous studies, conventional coordinated transmission schemes allocated a set or cluster of coordinated transmission SBSs based on SINR performance, energy efficiency, and control overhead. To the best of our knowledge, this paper is the first study that exploits resource utilisation to overcome service performance degradation owing to overloaded SBSs in coordinated transmission scenarios, in which there exist several hotspot areas with high service demand. The proposed coordinated transmission can achieve a significant increase in the number of UEs serviced by the network because the SBSs with more available resources are cooperatively involved in the coordinated transmission.

• In the case of incorrect channel estimation, we also derive robust resource allocation to avoid violating the minimum SINR even in the worst case. When we obtain the CSI for a coordinated transmission, the channel estimate can be inaccurate owing to interference from non-cooperative cells. In this case, UEs may fail to have high SINRs, which they are expected to obtain through coordinated transmission from cooperating SBSs. In this paper, a robust optimisation is considered for coordinated transmission in order to compensate for inaccurate channel estimates and to mitigate interference from non-cooperative cells.
We exploit user-centric clustering-based coordinated transmission to achieve a high level of coordination diversity and to enhance network throughput. The cooperating SBSs are determined based on possible clustering sets for each UE. We derive the outage probability for a user, and show that the proposed method provides robust and efficient service performance even with inaccurate CSI.

3 System model

We consider a wireless network where a set of N SBSs, denoted by \( X \), provide wireless connectivity service to a set of \( L \) UEs, denoted by \( U \), i.e., \( |X| = N \) and \( |U| = L \). As illustrated in Fig. 1, some SBSs are located in heavily congested areas with high demand for UE services, while the others are in less congested areas. These heterogeneous characteristics of wireless service capability and demand cause an unbalance of QoS and should be taken into consideration in a coordinated transmission policy. SBSs simultaneously provide wireless connectivity service for UEs by adopting orthogonal frequency division multiple access (OFDMA) with a set of subcarriers \( c \). A set of neighbouring SBSs for a UE \( k \), denoted by \( X_k \subseteq X \), is defined as those which can communicate with the UE \( k \). If the size of \( X_k \) is large, it is possible to exploit a high level of coordination diversity with more SBSs, but this approach incurs significant inter-SBS interference if the SBSs are not properly coordinated. The SBSs share transmission data and CSI with each other, and SBS \( i \) transmits a message \( m_{i,k} \) to UE \( k \) at a subcarrier \( c \). Note that the SBSs are connected to each other through a high-speed wired backhaul network. However, if the size of \( X_k \) is large, message exchange overhead may be significant. Among SBSs in \( X_k \), several SBSs, denoted by \( S_{k}(t) \subseteq X_k \), are selected to participate in joint signal transmission to UE \( k \) at subcarrier \( c \) at time slot \( t \). We define SBS resource utilisation \( u_i \) for the SBSs \( i \) as the ratio of the currently occupied subcarriers to the total number of subcarriers at time slot \( t \). For example, SBSs \( i \) is fully utilised if all \( c \) subcarriers are occupied owing to a high number of UEs, i.e., \( u_i = 1 \). If SBS \( i \) is idle without UEs, \( u_i = 0 \). In general, as illustrated in Fig. 1, the utilisations of SBSs in the heavily congested areas are high, while those of SBSs in the lightly congested areas are low, because the SBSs in the more congested areas receive higher wireless service demands from more UEs. This implies that the SBSs in less congested areas have more available resources to be allocated for new UEs in a coordinated transmission scheme.

In Fig. 1, the received signal of UE \( k \) at subcarrier \( c \) at time slot \( t \) is given by

\[
R_{j,k} = \sum_{i \in X} h_{i,k}(t)\sqrt{P_{i,k}}m_{i,k}(t) + n_{j,k}(t),
\]

where \( h_{i,k}(t) \) is the channel gain from SBS \( i \) to UE \( k \) at subcarrier \( c \), \( m_{i,k}(t) \) is a transmitted message from SBS \( i \), and \( n_{j,k}(t) \) is the additive white Gaussian noise (AWGN) with a variance of \( \sigma^2 \). From a UE \( k \)'s perspective, there is a set of SBSs, \( X_k \), which can communicate with UE \( k \) while the other SBSs in a network are only interferers. Thus the signal term in (1) can be decomposed into two components, which are the signals from the SBSs that are jointly cooperating for the wireless service to UE \( k \) and the interference from the other SBSs. Then the SINR can be easily derived as done in the literature [9, 10].

In this paper, we decompose \( R_{j,k} \) into three components as follows

\[
R_{j,k} = \sum_{i \in X_k} h_{i,k}(t)\sqrt{P_{i,k}}m_{i,k}(t) + \sum_{i \in X \setminus S_{k}(t)} h_{i,k}(t)\sqrt{P_{i,k}}m_{i,k}(t) + \sum_{i \in X_k} h_{i,k}(t)\sqrt{P_{i,k}}m_{i,k}(t) + n_{j,k}(t),
\]

(2)

where the above equation is equivalent to that in (1) because \( X = S_{k}(t) \cup (X \setminus S_{k}(t)) \cup (X' \setminus X_k) \). Under the proposed coordinated transmission scheme, the SBSs in \( (X \setminus S_{k}(t)) \) are controlled using precoding matrices so as to not interfere with UE \( k \) when the SBSs in \( S_{k}(t) \) are transmitting to UE \( k \). Therefore, SINR of UE \( k \) is obtained by using (2) as

\[
\text{SINR}_{j,k} = \frac{\sum_{i \in S_{k}(t)} h_{i,k}(t)^2 P_{i,k}}{\sum_{i \in X \setminus S_{k}(t)} h_{i,k}(t)^2 P_{i,k} + \sigma^2}
\]

(3)

\[
= \sum_{i \in S_{k}(t)} g_{i,k}(t),
\]

(4)

where \( g_{i,k}(t) = h_{i,k}(t)^2 P_{i,k} / (\sum_{i \in X \setminus S_{k}(t)} h_{i,k}(t)^2 P_{i,k} + \sigma^2) \) if the power of \( m_{i,k} \) is assumed to be 1. Note that the denominator of (3) does not include the interference from the SBSs in \( (X \setminus S_{k}(t)) \). Due to the SINR of UE \( k \) in (3) depends on the signal strengths from \( S_{k}(t) \), selecting a subset of appropriate SBSs \( S_{k}(t) \) from \( X_k \) is crucial for the throughput performance.

4 Robust coordinated transmission

4.1 Coordinated transmission

We consider a coordinated transmission that enables multiple BSs to cooperate with each other in order to improve the throughput performance of UEs. If a set of cooperating BSs \( X_k \) provides wireless connectivity service to UE \( k \) by making use of spatial diversity, the performance would be improved compared to the case where only one of the neighbouring BSs provides service to UE \( k \). However, message exchange overhead exists if the size of \( X_k \) is large because BSs in \( X_k \) must share transmission data and CSI. Instead of using the entire set of BSs, a subset of \( X_k \) needs to be selected to participate in joint signal transmission to UE \( k \). For the selection of \( S_{k}(t) \), one may consider to minimise the total transmit power of BSs in \( S_{k}(t) \) under the SINR requirement constraints as follows

\[
\text{minimise } \sum_{i \in S_{k}(t)} \sum_{j \in C} \sum_{c \in S_{k}(t)} P_{i,c}
\]

subject to \( \sum_{i \in S_{k}(t)} g_{i,k}(t) \geq \gamma \) for \( \forall c \in C \).

(5)

For simplicity, the aforementioned optimisation is decomposed for each subcarrier as follows: [Hereafter, we omit the subscript \( c \) for notational simplicity.]

**MIN-POWER method**

\[
\text{minimise } \sum_{i \in X_k} P_i
\]

subject to \( \sum_{i \in S_{k}(t)} g_{i,k}(t) \geq \gamma \).

(6)

Note that as addressed in Section 2, there exist several approaches [11, 12] that have used an optimisation strategy similar to (6), and JSPA [11] will be evaluated for comparison purposes in Section 5. However, because SBSs are usually powered by an electrical outlet, what is more important than power consumption in the selection of an SBS is the impact on the QoS of UEs served by an SBS when the SBS is newly selected to provide service to another UE.

We propose a new criterion for SBS selection, which is to select a subset of SBSs with low resource utilisation for joint signal transmission to UE \( k \) at subcarrier \( c \). Determining a subset of cooperative SBSs without considering their utilisation may lead to a degradation of service capability. An SBS with low utilisation can accept more service requests from UEs without degrading the QoS of ongoing services. On the other hand, if a few SBSs that are close to UEs are selected in a hotspot area, the SBSs are easily overloaded, and the QoS provided by the SBSs eventually
degrades. Instead of high-utilisation SBSs, it is desirable to select low-utilisation SBSs to improve the throughput performance of UE $k$ by cooperatively providing service to the UE. We determine $S_k(t)$ that minimises the sum of utilisation $u_i(t)$, $i \in S_k(t)$ while satisfying the SINR threshold requirement. The selection problem is formulated as follows.

**MIN-UTIL method**

\[ \begin{align*}
\min_{S_k(t)} & \quad \sum_{i \in S_k(t)} u_i(t) \\
\text{s.t.} & \quad \sum_{i \in S_k(t)} g_{ik}(t) \geq \gamma
\end{align*} \tag{7} \]

UE $k$ is served at time slot $t$ by the SBSs in $S_k(t)$ obtained from (7).

We further extend the MIN-UTIL optimisation in (7) to robust optimisation in order to compensate for uncertainty in parameter estimation. In (7), $g_{ik}(t)$ is a parameter to be estimated, and it is susceptible to interference from non-cooperative SBSs (i.e. $X\setminus X_k$) for UE $k$. If the SINR constraint is not satisfied owing to estimation uncertainty, the joint transmission by the selected SBSs is unnecessarily wasted because UE $k$ does not have an SINR high enough for successful communication. While $g_{ik}(t)$ is a nominal value to be estimated and used in the optimisation, the actual value of the uncertain parameter $\tilde{g}_{ik}(t), j \in S_k(t)$ is assumed to be in the range of $[g_{ik}(t) - \bar{g}_{ik}(t), g_{ik}(t) + \bar{g}_{ik}(t)]$ [16]. The robust optimisation for (7) is formulated as follows:

**ROBUST-MIN-UTIL method**

\[ \begin{align*}
\min_{S_k(t)} & \quad \sum_{i \in S_k(t)} u_i(t) \\
\text{s.t.} & \quad \sum_{i \in S_k(t)} g_{ik}(t) - \beta_{S_k(t)}(\Gamma) \geq \gamma
\end{align*} \tag{8} \]

where $\beta_{S_k(t)}(\Gamma) = \max_{i \in S_k(t), j \in X \setminus X_k} \{ \sum_{i \in S_k(t)} \tilde{g}_{ik}(t) \}$. In (8), the robustness of the system is adjusted by an integer parameter $\Gamma$, $0 \leq \Gamma \leq |S_k(t)|$. This implies that the robust solution obtained by (8) does not violate the SINR constraint if the number of $\tilde{g}_{ik}(t)$’s that have a significant discrepancy relative to the corresponding nominal values does not exceed $\Gamma$. Therefore the SBSs obtained from (8) provide more robust service to the UE than those obtained from (7) when an uncertainty in parameter estimation exists, which is always the case in practice.

### 4.2 Algorithm for a binary selection problem

The optimisation of (8) is a binary selection problem that minimises the aggregate utilisation of the selected SBSs while satisfying the SINR constraint. We first transform the minimisation problem to an equivalent maximisation form as follows.

\[ \begin{align*}
\max_{Z_k} & \quad \sum_{i \in Z_k} u_i(t) \\
\text{s.t.} & \quad \sum_{i \in Z_k} g_{ik}(t) \leq \sum_{i \in X_k} g_{ik}(t) - \gamma - \beta_{Z_k}(\Gamma). \tag{9} \end{align*} \]

Note that the maximisation of the aggregate utilisation of SBSs in $Z_k$ is equivalent to the minimisation of the aggregate utilisation of SBSs in $S_k(t)$. One may solve this binary integer programming problem using a brute-force search, but its complexity is given by $O(2^{|Z_k|})$, which is too expensive to be solved in practice.

Owing to the high complexity of (9), we solve the problem in two steps and obtain a suboptimal solution. In the first step, we select a set of SBSs by setting $\beta_{S_k(t)}(\Gamma) = 0$, which corresponds to the solutions of the MIN-UTIL method in (7). In the second step, more SBSs are selected from the set of SBSs that are not selected in the first step in order to compensate for the uncertainty in parameter estimation. Note that $\beta_{S_k(t)}(\Gamma)$ is approximated as a constant, which is obtained using the solution obtained in the first step.

In each step, we use dynamic programming in order to find a solution for the binary selection problem [17–19]. Algorithm 1 (see Fig. 2) shows the procedure of the dynamic programming used for solving (9). The algorithm is based on the tabulation of maximum values of the aggregate utilisation. Table 1 shows the tabulation of a parameter $\mathcal{V}(i, g_{ik})$, which represents the maximum value of the aggregate utilisation that can be attained using SBSs up to $i$ under the constraint that the summation of SINR is less than or equal to $\bar{g}_{ik}$. In this algorithm, $g_{ik}(t)$ must be normalised for the tabulation in Table 1 such that the normalised version $\tilde{g}_{ik}$ of $g_{ik}$ has a certain large integer value, which corresponds to the width of the table. That is, $\tilde{g}_{ik} = [g_{ik}(t)/(\epsilon g_{ik}(t)/|X_k|)]$ and
Algorithm 1

1: // For ∀i, i ∈ {0, · · · , x} where x is the number of SBSs.
2: // For ∀g′′k, g′k ∈ {0, · · · , y} where y is the maximum bound of normalised SINR sum.
3: // g′′k+1 is the normalised SINR value from SBS i to UE k.
4: // u(t) is the utilisation value of SBS i.
5: // Set V (i, 0) = 0 for ∀i, and set V (0, g′′k) = 0 for ∀g′′k.
6: // For ∀(i, g′′k), calculate V (i, g′′k) as follows:
7: if g′′k > g′k then
8: V (i, g′′k) = V (i − 1, g′k)
9: else
10: V (i, g′′k) = max{V (i − 1, g′k), V (i − 1, g′k − g′′k) + u(t)}
11: end if
12: end while
13: Let i = x, and g′′k = y.
14: while i > 0 and g′′k > 0 do
15: if V (i, g′′k) = V (i − 1, g′k) then
16: mark the ith SBS
17: g′′k = g′k − g′′k, i = i − 1
18: else
19: i = i − 1
20: end if
21: end while

The complexity of Algorithm 1 (Fig. 2) depends on the table size for the binary selection problem [18]. As the width and height of Table 1 are π = (|X| + 1) and θ = (|X| + 1) respectively, the complexity is given by O(|X|θ), which is much lower than O(2|X|) for the brute-force search. In comparison with the proposed algorithm, the algorithms in [11, 12] have a different approach to reduce the computational complexity. In [11], the JSPA algorithm reduces the number of possible cases for BS selection by restricting the number of cooperative BSs per each UE to two. Since UEs are provided wireless services from less than or equal to two BSs, the complexity of the JSPA algorithm is given by O(|X|2). In [12], BSs are statically clustered into groups with several BSs, and the optimisation is applied to each group of BSs rather than to the whole BSs for reducing the number of possible cases in BS selection. The complexity of the proposed algorithm using the dynamic programming is lower than that of the algorithms in [11, 12].

4.3 Outage probability

In this section, we calculate the upper bound of the outage probability for the ROBUST-MIN-UTIL method.

Theorem 1: Let S′′k(t) and A∗ be an optimal solution of (8) and the corresponding set that achieves the maximum for β(S′′k(t)), respectively. Then the outage probability satisfies the following inequality

\[ \Pr \left( \sum_{i \in S_k} \tilde{g}_{ik}(t) < h \right) \leq 1 - \Phi \left( \frac{\gamma - 1}{\sqrt{|X_k|}} \right) \]  

where \( \Phi(\cdot) = (1/\sqrt{2\pi}) \int_{-\infty}^{\theta} \exp(-y^2/2)dy \) and is the cumulative distribution function of a standard normal distribution.

Proof: See the Appendix.

The outage probability is calculated under the condition that a feasible solution exists. Note that there is no feasible solution if the SINR is lower than γ for S′′k(t) = X_k.

\[ \gamma = \sqrt{\epsilon \left( (\epsilon g_{X_k,t}^{\max}/|X_k|) \right)} \quad \text{for } \epsilon > 0 \]

Here, \( \epsilon \) is set to 0.1. Note that the SBSs are assumed to be sorted in ascending order of \( g_{X_k,t}(t) \). Let \( S_k^f \) and \( S_k^g \) be the solution for the first and second steps, respectively. \( S_k^f \) is obtained using Algorithm 1 (Fig. 2) with \( x = |X_k| \) and \( y = \sum_{i \in X_k} g_{ik} - \gamma \). In Algorithm 1 (Fig. 2), the table for the parameter \( V(i, g_k) \) is initialised in line 6. In lines 7–12, each element \( V(i, g_k) \) of the table is recursively calculated as follows

\[ V(i, g_k) = \begin{cases} 
V(i - 1, g_{k}'), & \text{if } g_{k}' > g_{k}'', \\
\max\{V(i - 1, g_k'), V(i - 1, g_{k}' - g_{k}'') + u(t)\} & \text{otherwise.}
\end{cases} \]

(10)

Using the table for \( V(i, g_k) \) as shown in Table 1, the algorithm starts to inspect each element of the table starting from the bottom right corner of the table, and if the condition in line 16 is satisfied, it sets the corresponding SBS as marked. The algorithm stops when all the rows are inspected, as described in lines 15–22. Eventually, the marked SBSs are selected as the SBSs that maximise \( \sum_{i \in Z_k} u(t) \), and \( S_k^f = (X_k \setminus Z_k) \).

Once \( S_k^f \) is obtained, \( S_k^g \) can be obtained by choosing SBSs from \( Z_k \) in order to compensate for the uncertainty in parameter estimation. In (9), \( \beta(S_{X_k,t}(\Gamma)) \) is a function of \( S_k(t) \), but is assumed to be a constant given by \( \beta(S_{X_k,t}(\Gamma)) \). Then \( S_k^g \) can be obtained simply by repeating Algorithm 1 (Fig. 2) with \( x = |Z_k| \) and \( y = \sum_{i \in Z_k} g_{ik} - \beta \). Note that the SBSs in \( Z_k \) are sorted in ascending order of \( g_{ik} \). Then \( g_{ik}' = [g_{ik}(t)/(\epsilon g_{X_k,t}(\Gamma)/|X_k|)] \), and \( \beta = [\beta(S_{X_k,t}(\Gamma))/(\epsilon g_{X_k,t}(\Gamma)/|X_k|)] \) for \( \epsilon > 0 \). Finally, the SBSs in \( S_k^f \cup S_k^g \) are intended to cooperatively provide service to UE k.

Table 1 Table for solving the binary selection problem using dynamic programming

<table>
<thead>
<tr>
<th>( g_k = 0 )</th>
<th>( g_k = 1 )</th>
<th>( g_k = 2 )</th>
<th>( \cdots )</th>
<th>( g_k = y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 0 )</td>
<td>( V(0,0) )</td>
<td>( V(0,1) )</td>
<td>( V(0,2) )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( i = 1 )</td>
<td>( V(1,0) )</td>
<td>( V(1,1) )</td>
<td>( V(1,2) )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>( i = x )</td>
<td>( V(x,0) )</td>
<td>( V(x,1) )</td>
<td>( V(x,2) )</td>
<td>( \cdots )</td>
</tr>
</tbody>
</table>
can easily become overloaded because they service excessive UEs. Experiences low outage performance, SBSs that are close to UEs method. Even though the SINR margin increases and each UE method is still lower than that of the ROBUST-MIN-UTIL uncertainty. When the SINR margin for protection against channel estimation probability. Note that the JSPA method in [11] does not consider performance of the ROBUST-MIN-UTIL method is always better than that of the other methods because the small SINR margins of performance comparison between the methods (|X| = 45, |X_k| = 30) On the other hand, the performance of the MIN-UTIL method with α = 0.12 increases as γ increases until γ = 17, and the MIN-UTIL method shows better performance than the ROBUST-MIN-UTIL method when γ ≥ 17. Since both the ROBUST-MIN-UTIL and the MIN-UTIL methods make use of low-utilisation SBSs to avoid QoS degradation of ongoing services caused by overloaded SBSs, the performance is highly dependent on the amount of SINR margin. Hence, when β_{MIN} (Γ) of the ROBUST-MIN-UTIL method is smaller than the SINR margin αβ of the MIN-UTIL method, the performance of the ROBUST-MIN-UTIL method is lower than that of the MIN-UTIL method. However, the performance of the MIN-UTIL method also eventually decreases, as shown in the simulation results, when γ ≥ 17. This is because excessive SBSs are selected to maintain the outage performance of UEs. As a result, the total number of UEs in service decreases. Moreover, when γ ≥ 17, the MIN-UTIL method consumes more resources than the ROBUST-MIN-UTIL method. Fig. 4b shows the simulation results when the number of UEs is 120. In this case, the performance comparison between the ROBUST-MIN-UTIL and the MIN-UTIL methods is similar to the results in Fig. 4a, except that the performance of both methods decreases more quickly as γ increases. However, unlike the simulation results in Fig. 4a, both the ROBUST-MIN-UTIL and the MIN-UTIL methods always show better performance than the MIN-POWER method. The MIN-POWER method selects SBSs that are close to UEs even if they are highly utilised. Hence, as the number of UEs increases, SBSs that are close to UEs are repeatedly selected and become overloaded, causing degradation in
the QoS. On the other hand, the ROBUST-MIN-UTIL and the MIN-UTIL methods make use of low-utilisation SBSs, since they select SBSs in a way that minimises the utilisation sum of selected SBSs. Therefore the ROBUST-MIN-UTIL and the MIN-UTIL methods show better performance than the MIN-POWER method, especially in a hotspot area with many UEs.

Fig. 5 shows the average transmit power and utilisation of SBSs for the simulation in Fig. 4a. Note that the SINR margins for both the MIN-POWER and the MIN-UTIL methods increase as $\alpha$ increases because the SINR margin is calculated as $\alpha \gamma$. Fig. 5a shows that when $\alpha = 0.02$, both the MIN-POWER and the MIN-UTIL methods require lower transmit power than the ROBUST-MIN-UTIL method. In these cases, the outage probability of the MIN-POWER and the MIN-UTIL methods is better than that of the ROBUST-MIN-UTIL method since the SINR margin is too small to compensate for the channel estimation uncertainty. Therefore the MIN-POWER and MIN-UTIL methods service fewer UEs, as shown in Fig. 4a. When $\alpha = 0.12$, there are points at which the MIN-POWER and MIN-UTIL methods consume more power than the ROBUST-MIN-UTIL method because the SINR margin $\alpha \gamma$ becomes larger than $\beta_s(T)$ in the ROBUST-MIN-UTIL method. When $\alpha = 0.12$ and $\gamma = 20$, however, even though the MIN-POWER method consumes more power than the ROBUST-MIN-UTIL method, the performance of the ROBUST-MIN-UTIL method is better than that of the MIN-POWER method. On the other hand, the MIN-UTIL method shows better performance than the ROBUST-MIN-UTIL method when $\alpha = 0.12$ and $\gamma \geq 17$.

However, the MIN-UTIL method requires much more power than the ROBUST-MIN-UTIL method, as shown at $\gamma = 20$. Furthermore, when $\gamma = 14$, although the ROBUST-MIN-UTIL and MIN-UTIL methods consume similar average transmit power, the ROBUST-MIN-UTIL method services more UEs than the MIN-UTIL method. This is because, despite the fact that the two methods have the same SINR threshold, the number of required SBSs changes from time to time owing to the difference in interference from non-cooperative SBSs. Hence, deriving the SINR margin from the selected SBSs is better than a static calculation of the SINR margin with regard to $\gamma$. Note that the JSPA method in [11] limits the number of UEs that are served by multiple SBSs in order to reduce the power consumption. As a result, the JSPA method consumes much less power than the other methods, but it provides wireless service to a smaller number of UEs than the other methods.

Figs. 5b show the average utilisation of SBSs that are selected to serve UEs. When the SINR margin is small, the MIN-UTIL method shows lower utilisation than the ROBUST-MIN-UTIL method. When the SINR margin is large, there are certain points at which the ROBUST-MIN-UTIL method shows lower utilisation than the MIN-UTIL method. On the other hand, according to the simulation results, the MIN-POWER method always shows higher utilisation than both the ROBUST-MIN-UTIL and the MIN-UTIL methods. This is because the ROBUST-MIN-UTIL and MIN-UTIL methods select low-utilisation SBSs instead of high-utilisation SBSs, while the MIN-POWER method selects SBSs that are close to UEs even if their utilisations are high. Therefore the ROBUST-MIN-UTIL and the MIN-UTIL methods can avoid QoS degradation of ongoing services more effectively than the MIN-POWER method. The JSPA method shows constantly low utilisation because the number of UEs that are served by multiple SBSs is limited in the JSPA method.

Fig. 6 shows the Jain’s fairness index for the utilisation of SBSs. The result of the Jain’s fairness index ranges from $1/|X|$ to 1, and the $1$ means that the utilisations of all SBSs are the same. The results indicate that the ROBUST-MIN-UTIL and the MIN-UTIL methods are better than the MIN-POWER method and the JSPA method in terms of fairness. This means that a few SBSs are excessively selected when the MIN-POWER method or the JSPA method is used, even though there are other SBSs with low utilisation. On the other hand, the ROBUST-MIN-UTIL and the MIN-UTIL methods select low-utilisation SBSs instead of high-utilisation SBSs. Therefore, the ROBUST-MIN-UTIL and the MIN-UTIL methods can avoid QoS degradation caused by SBSs with high utilisation.

6 Conclusion

In this paper, we have proposed a robust optimisation-based SBS selection method in which low-utilisation SBSs are selected to
cooperatively provide service to UEs while compensating for uncertainty in parameter estimation. The proposed method attempts to minimise the utilisation of neighbouring SBSs and avoid service performance degradation caused by overloaded SBSs. It also compensates for the uncertainty in parameter estimation for robust coordination transmission with low outage probability. We have derived the upper bound of outage probability and compared the upper bound with simulation results. The simulation results indicated that the proposed method achieves efficient and robust service performance in a dense small cell network.

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9 Appendix: Proof of Theorem 1
We define \( \eta_i(t) = \left( \hat{g}_{i,k}(t) - g_{i,k}(t) \right)/\hat{g}_{i,k}(t) \) and \( \eta_i(t) \in [-1, 1] \).

\[
\Pr \left( \sum_{i \in S_k(t)} \eta_i(t) < \gamma \right)
= \Pr \left( \sum_{i \in S_k(t)} g_{i,k}(t) + \sum_{i \in S_k(t)} \eta_i(t) \hat{g}_{i,k}(t) < \gamma \right)
\leq \Pr \left( \sum_{i \in S_k(t)} g_{i,k}(t) + \sum_{i \in S_k(t)} \eta_i(t) \hat{g}_{i,k}(t) < \gamma \right)
< \Pr \left( \sum_{i \in S_k(t)} g_{i,k}(t)N(t) - \beta g_{i,k}(t) \right)
= \Pr \left( \sum_{i \in S_k(t)} \eta_i(t) \hat{g}_{i,k}(t) < - \sum_{i \in A^c} \hat{g}_{i,k}(t) \right)
\leq \Pr \left( \sum_{i \in S_k(t)} \eta_i(t) \hat{g}_{i,k}(t) < - \sum_{i \in A^c} \left(1 - \eta_i(t) \hat{g}_{i,k}(t) \right) \right)
= \Pr \left( \sum_{i \in S_k(t)} \hat{g}_{i,k}(t) - \beta g_{i,k}(t) \eta_i(t) < \sum_{i \in A^c} \eta_i(t) - \Gamma \right)
= \Pr \left( \Gamma < \sum_{i \in S_k(t)} \hat{g}_{i,k}(t) \eta_i(t) \right),
\] (12)
where
\[
\nu_i = \begin{cases} 1, & \text{if } i \in A, \\ \frac{\hat{g}_{i,k}(t)}{\beta g_{i,k}(t)}, & \text{if } i \in S_k(t) \setminus A, \end{cases}
\] and \( \alpha = \arg \min \eta_i(t) \). By using the theorem in [16], the probability in (12) is less than or equal to \( 1 - \Phi((1-1/\sqrt{|S_k(t)|})) \).