A Data-Driven Indirect Estimation of Machine Parameters for Smart Production Systems

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***Abstract*—Automated measurement of the machine reliability parameters for a production system enables a continuous update of the mathematical model of the system, which can be used for various analyses toward productivity improvement. However, the continuous update may be impeded by some machines of which automated parameter measurements are out of order. Such a situation has been observed, for instance, when some of the machines in the line cannot save log files or Internet of Things devices that measure these machines stop functioning. In this context, this article addresses the problem of estimating the reliability parameters of those machines while avoiding a direct manual measurement (by humans) of uptime and downtime. It turns out that those parameters can be computed using buffer-related data of the neighboring machines along with the system information. With this, a continuous update of the model is possible even though some machines stop recording their status in an automated manner. The method is indirect as opposed to direct manual measurement. The results are derived for synchronous serial production lines with Bernoulli and also exponential reliability characteristics. Our simulation studies verify the accuracy of the proposed estimation methods.**

***Index Terms*—Data-driven, Industry 4.0, parameter estimation, smart factory, smart production systems.**

I. Introduction

ADVANCES of Internet of Things (IoT) and cyber–physical systems technologies have accelerated the development of smart factories, where real-time data from the factory floor are collected and utilized to manage the whole operation for improved productivity, lower level of inventories, lower energy consumption, and on-demand manufacturing, etc. [1]–[4].

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Two categories of technologies seem necessary to enable smart factories: the first is an infrastructure for data acquisition and utilization, and the second is a set of methods to address specific problems that are of importance.

In the first category, several data system frameworks have been developed (e.g., Arrowhead [5], Falcon [6], Reference Model by Industrial Data System Association [7], [8], etc.) that facilitate data collection, storage, information exchange, and request/provide analytics-based services. The second category is traditionally an area for industrial engineering, where queuing theory, linear programming, decision theory, etc., have been developed [9].

As one of the constituents of technologies in the second category, *smart production systems* has been a topic of active research [10]–[12]. Smart production systems address the problem of productivity improvement using the work of production systems engineering (PSE) [13], which provides a mathematical foundation to solutions of many important problems of production systems such as bottleneck identification, lean buffering, customer demand satisfaction, etc. See [13] for more details. The theory of PSE develops a mathematical model of the production line, all parameters of which are computed using the data measured from the factory floor, and uses the model for various analyses. Key parameters are mean time to repair (MTTR) and mean time between failures (MTBF) for each machine in the production system. Those two parameters are important since machine reliability characteristics are defined based on the two parameters. Estimating MTBF and MTTR requires observation of many instances of machine down, which takes an extended period of time. The use of IoT devices is expected to significantly reduce the effort required for collecting such data. Recently, the number of uptime/downtime measurements, necessary to ensure a given accuracy and likelihood of MTTR and MTBF estimates, was derived with mathematical rigor in [14]. A modeling approach distinct from measuring uptime and downtime of each machine is proposed in [15] where measurement of work in process for each buffer and the throughput of the line are used instead.

Eventually, the first and the second categories must be integrated so that the first will feed real-time data to the second so that results of analytics are continuously updated. While such integration is slowly progressing, some effort has been made to analyze productivity of a production line based on existing (rather than newly proposed as in [5]–[8]) fault monitoring data infrastructure [16]. As it turns out, situations arise in production systems where some of the machines fail to report status to a data system. Indeed, the authors of this manuscript encountered manufacturing lines in Daegu Metropolitan City, Korea, which have been implemented with data logging systems under the government initiative for the smart factory, but some of the logging functions are not operational. Repair has been avoided since it is viewed as costly both in time and resource.

The motivation of this work is as follows. Such a situation hinders continuous updates on machine reliability parameters. The modeling of the entire production system cannot be completed due to a machine whose logging is not operational. We refer to such a machine as *unknown machine* in the sense that the data from the machine are not available. Estimating the reliability parameter of an unknown machine is an important problem. As mentioned before, an unknown machine impedes modeling of the production line and consequently productivity analysis.

In this work, we consider the problem of estimating key parameters (MTTR and MTBF) of an unknown machine. To the best of our knowledge, existing work does not address this problem. Most of the existing work assumes all data are available. The methods are developed for synchronous serial lines that follow the Bernoulli model and the exponential reliability model, respectively. We refer to the method developed in this work as "indirect estimation" in the sense that the reliability parameters of a machine are obtained not by direct measurement of the uptime/downtime of the machine but by estimation from the data of the other machines in the line.

The main contributions of this article are as follows.

1) A method of estimating the reliability parameter of an unknown machine in a production line that follows Bernoulli model using the data from neighboring machines.
2) A method of estimating the reliability parameters of an unknown machine in a production line that follows synchronous exponential model using the data from neighboring machines.
3) It is shown that the methods are applicable to a production line with more than one unknown machines as long as they are not placed consecutively.
4) Evaluation of the estimation accuracy using simulation is performed to show that proposed methods are practical.

In addition, the proposed methods in this article may serve as a feature that enhances the resiliency of smart production systems because they provide means to deal with failures of some sensors or circuits in the data system with no additional costs to repair them. This may be viewed as technologies in the second category operate in a way to compensate the deficiency in the first category, i.e., data systems.

The outline of this article is as follows. Section II describes synchronous serial production lines with machines and buffers. Descriptions of the performance measures are also given. Sections III and IV develop an indirect method for lines with Bernoulli reliability characteristics and illustrate the accuracy. Sections V and VI carry out a similar development for lines with exponential reliability characteristics. Finally, Section VII concludes the article.

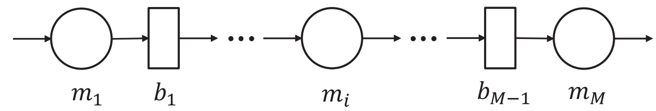

Fig. 1. Standard form of serial production line.

II. Synchronous Serial Production Lines, Performance Measures, and Unknown Machines

A serial production line is shown in Fig. 1 which consists of M machines $(m_1, \ldots, m_N)$ and $M-1$ buffers $(b_1, \ldots, b_{M-1})$. All machines work synchronously, i.e., all machines are assumed to have constant and identical cycle times denoted by τ. The capacity of buffer b_i is denoted by a positive integer N_i.

Two types of reliability models are considered in this article: the Bernoulli reliability model for slotted time case and the exponential reliability model for continuous-time case. These two models permit analytical results for production system analysis while other reliability models do not. More importantly, many production systems are well approximated by these two models. Many examples exist that adopted these models to analyze actual production systems [17]–[20]. Reliability parameter for the former is p_i, i.e., the probability that a machine stays up for each duration of τ. This parameter for the machine m_i is typically obtained by

$$p_i = \frac{T_{\text{up},i}}{T_{\text{up},i} + T_{\text{down},i}} \tag{1}$$

where $T_{\text{up},i}$ is the average uptime (MTBF) obtained from uptime measurement and $T_{\text{down},i}$ is the average downtime (MTTR) obtained from downtime measurement.

For the exponential lines, the reliability of the ith machine is described by two parameters, the failure rate λ_i and the repair rate μ_i. The two are typically obtained from the measurement by

$$\lambda_i = \frac{1}{T_{\text{up},i}}, \quad \mu_i = \frac{1}{T_{\text{down},i}}. \tag{2}$$

Some performance measures of the production lines are introduced.

1) Production rate (PR): The average number of parts produced by the last machine per cycle time in the steady-state.
2) Starvation ratio of m_i (ST_i): The steady-state probability that m_i stops working due to an empty upstream buffer.
3) Blockage ratio of m_i (BL_i): The steady-state probability that m_i stops working due to a full downstream buffer.

In this article, we assume the machines in the production lines have automated data measurement systems. They record uptime and downtime for MTBF and MTTR and starvation/blockage time for ST_i and BL_i. Additionally, two buffer state indicators are also measured: one that records if the upstream buffer is empty or not, and the other that records if the downstream buffer is full or not. If a machine m_i is an *unknown machine*, none of the data from the machine such as $T_{\text{up},i}$, $T_{\text{down},i}$, ST_i, and BL_i is available.

It must be pointed out that the number of observation for relevant events (e.g., machine failure, buffer full or empty, etc.) affects the estimation accuracy. The problem of ensuring a given accuracy has been rigorously investigated in [14]. Thus, in this work, we assume the estimates are based on large enough data points to ensure the desired accuracy.

A line may have one or multiple unknown machines. For the unknown machine m_i, the reliability parameter p_i for Bernoulli line needs to be obtained by a means other than (1) and the reliability parameters λ_i and μ_i for the exponential line by a means other than (2). We intend to develop such methods.

III. Indirect Estimation for Bernoulli Lines

A. Analysis of Performance Measures

How the performance measures relate to all other parameters in the system is briefly reviewed here based on the work of [13]. Exact solutions for performance measures exist only for the two-machine serial line. In order to analytically calculate the performance measures, the production line is modeled as a Markov chain, of which the states are the buffer occupancy. For real numbers $0 < x, y < 1$ and a natural number N, define the function

$$Q(x,y,N) = \begin{cases} \dfrac{(1-x)(1-\alpha)}{1-\frac{x}{y}\alpha^N}, & \text{if } x \neq y, \\ \dfrac{1-x}{N+1-x}, & \text{otherwise} \end{cases}$$

with $\alpha = \frac{x(1-y)}{y(1-x)}$. The function $Q(x,y,N)$ represents the probability that the buffer is empty. Then, the performance measures of the two-machine line are given by

$$\begin{aligned} \mathrm{PR} &= p_1(1-Q(p_2,p_1,N_1)) \\ &= p_2(1-Q(p_1,p_2,N_1)), \end{aligned} \tag{3}$$

$$\mathrm{BL}_1 = p_1 Q(p_2,p_1,N_1), \tag{4}$$

$$\mathrm{ST}_2 = p_2 Q(p_1,p_2,N_1). \tag{5}$$

For the production lines with $M > 2$, the Markov chain approach is not practical because the number of states is the multiple of all possible states of each buffer, i.e., $\Pi_{i=1}^{M-1} N_i$, which increases exponentially as the number of buffers increases. Alternatively, the following aggregation method is used. The aggregation method approximates the entire line by a virtual two-machine line. The total of $M-1$ choices of the virtual two-machine line exists, each of which is anchored at a buffer. For instance, all machines and buffers before the buffer b_i are approximated by the first virtual machine, and all after the buffer b_i are approximated by the second virtual machine. This is depicted in Fig. 2. The forward Bernoulli parameter for m_i^f is expressed as p_i^f, and the backward Bernoulli parameter of m_{i+1}^b is expressed as p_{i+1}^b.

The forward and backward parameters of each machine are defined recursively with an iteration index $s = 0, 1, 2, \ldots,$ as follows:

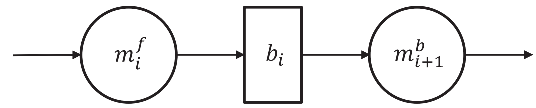

Fig. 2. Two-machine serial line by the aggregation method, $i = 1, 2, \ldots, M-1$.

$$\begin{aligned} p_i^b(s+1) &= p_i(1-Q(p_{i+1}^b(s+1), p_i^f(s), N_i)), \\ &\quad i = 1, \ldots, M-1, \\ p_i^f(s+1) &= p_i(1-Q(p_{i-1}^f(s+1), p_i^b(s+1), N_{i-1})), \\ &\quad i = 2, \ldots, M \end{aligned} \tag{6}$$

with $p_i^f(0) = p_i, \quad i = 1, \ldots, M$ as initial conditions and $p_1^f(s) = p_1, \ p_M^b(s) = p_M$, for all s as boundary conditions. Parameters $p_i^f(s)$ and $p_i^b(s)$ converge in the procedure of (6) and yield

$$p_i^b := \lim_{s\to\infty} p_i^b(s), \ p_i^f := \lim_{s\to\infty} p_i^f(s). \tag{7}$$

Using the aggregation method, the performance measures of the serial lines with $M > 2$ are given by

$$\begin{aligned} \widehat{\mathrm{PR}} &= p_1^b = p_M^f \\ &= p_{i+1}^b(1-Q(p_i^f, p_{i+1}^b, N_i)) \\ &= p_i^f(1-Q(p_{i+1}^b, p_i^f, N_i)), \ i = 1, \ldots, M-1, \end{aligned} \tag{8}$$

$$\widehat{\mathrm{BL}_i} = p_i Q(p_{i+1}^b, p_i^f, N_i), \ i = 1, \ldots, M-1, \tag{9}$$

$$\widehat{\mathrm{ST}_i} = p_i Q(p_{i-1}^f, p_i^b, N_{i-1}), \ i = 2, \ldots, M. \tag{10}$$

Since these measures are obtained by an approximation, they are denoted with "hat" to indicate this nature.

B. Two-Machine Lines ($M = 2$)

When one of the two machines is an unknown machine, its Bernoulli parameter can be calculated using (4) or (5), which is formulated in the following theorem.

Theorem 1: Consider the production line shown in Fig. 1 with $M = 2$. Let m_i be the unknown machine.

1) If $i = 1$, then p_1 is given by the solution x of the following equation:

$$Q(x, p_2, N_1) = \frac{\mathrm{ST}_2}{p_2}. \tag{11}$$

2) If $i = 2$, then p_2 is given by the solution x of the following equation:

$$Q(x, p_1, N_1) = \frac{BL_1}{p_1}. \tag{12}$$

Proof: See Appendix. ■

Remark 1: In regard to solving (11) or (12) for x, one always encounters an invalid solution that needs to be discarded. When solving for x the equation $Q(x, p, N) = \xi$ with given

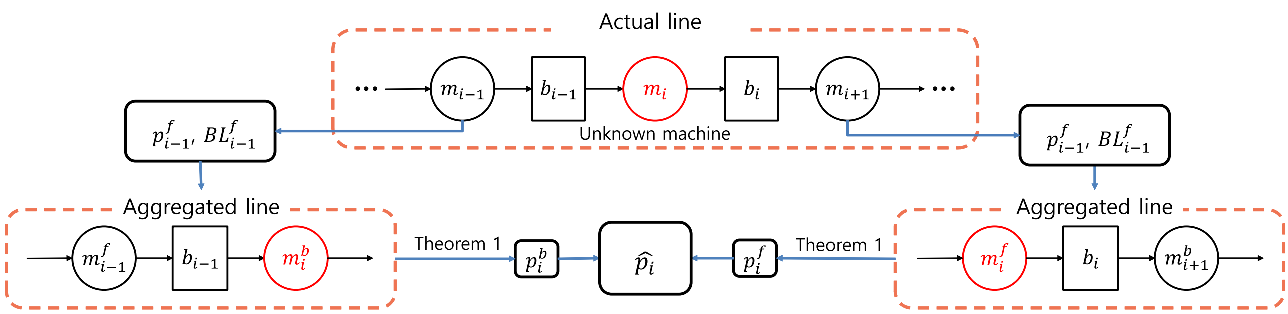

Fig. 3. Procedure for data-driven indirect estimate method of Bernoulli production lines.

$0 < p, \xi < 1$, and a natural number N, the mathematical derivation using the formula of Q for $x \neq p$ always yields that $x = p$ is a solution regardless of ξ. However, this is not a valid one since the formula is for $x \neq p$. Using a bisection algorithm to solve (11) and/or (12) effectively avoids the invalid solution.

The implication of the results is as follows. If m_1 is the unknown machine, i.e., $T_{\text{up},1}$ and $T_{\text{down},1}$ are not available, p_1 cannot be computed by (1). However, p_1 can be obtained by Theorem 1 using ST_2. Note that, as mentioned in Section I, ST_2 can be computed using the log of m_2. The same applies for the case when m_2 is the unknown machine, i.e., BL_1 can be computed from the log of m_1.

C. Lines With More Than Two Machines ($M > 2$)

When $M > 2$, the results of aggregation method, i.e., (6), (9), and (10), are used for the data-driven indirect estimation.

Theorem 2: Consider the production lines shown in Fig. 1 with $M > 2$. Let m_i be an unknown machine.

1) If $i \in \{2, \ldots, M-1\}$, then, $\hat{p}_i$ is given by

$$\hat{p}_i = \frac{p_i^b}{1 - Q(p_{i+1}^b, p_i^f, N_i)} \tag{13}$$

where p_i^b and p_i^f are the unique solutions, respectively, of the following equations in x

$$Q(x, p_{i-1}^f, N_{i-1}) = \frac{\mathrm{BL}_{i-1}}{p_{i-1}}, \tag{14}$$

$$Q(x, p_{i+1}^b, N_i) = \frac{\mathrm{ST}_{i+1}}{p_{i+1}}. \tag{15}$$

Here, p_{i-1}^f in (14) is given by $p_{i-1}^f = p_{i-1} - \mathrm{ST}_{i-1}$, and p_{i+1}^b in (15) by $p_{i+1}^b = p_{i+1} - \mathrm{BL}_{i+1}$.
Alternatively, $\hat{p}_i$ in (13) can also be obtained by

$$\hat{p}_i = \frac{p_i^f}{1 - Q(p_{i-1}^f, p_i^b, N_{i-1})}. \tag{16}$$

2) If $i = 1$, then $\hat{p}_1$ is given by the solution x of the following equation:

$$Q(x, p_2^b, N_1) = \frac{\mathrm{ST}_2}{p_2} \tag{17}$$

where $p_2^b = p_2 - \mathrm{BL}_2$.

3) If $i = M$, then, $\hat{p_M}$ is given by the solution x of the following equation:

$$Q(x, p_{M-1}^f, N_{M-1}) = \frac{\mathrm{BL}_{M-1}}{p_{M-1}} \tag{18}$$

where $p_{M-1}^f = p_{M-1} - \mathrm{ST}_{M-1}$.

Proof: See Appendix ∎

Insight for obtaining the estimates in Theorem 2 is given in Fig. 3. As seen in 1), for the estimation, the information of the machines and buffers located before and after the unknown machine is necessary. Note that the result is applicable when unknown machines are multiple, as long as they are not consecutive.

IV. Accuracy of Estimation on Bernoulli Lines

A. Model Validation With Estimated Parameters

The estimation of Bernoulli parameter for the $M > 2$ case is not exact because it is based on the aggregation procedure, which is an approximation. Here, we investigate the accuracy of the proposed method with two metrics

$$\epsilon_p = \frac{|p_i - \hat{p}_i|}{p_i}, \ \epsilon_{\mathrm{PR}} = \frac{|\mathrm{PR}_s - \mathrm{PR}_i|}{\mathrm{PR}_s} \tag{19}$$

where PR_s is the production rate obtained by the simulation, and PR_i is the production rate obtained by the aggregation method with m_i being unknown. The former shows the accuracy of $\hat{p}_i$, and the latter shows how closely the model predicts PR with the estimated parameter for the unknown machine.

For the investigation, we consider production lines with $M =$ 5, 10, 15, and 20. For each case, the parameters are randomly selected in the following ranges:

$$p_i \in [0.7, 1], \ N_i \in [1, 10]. \tag{20}$$

The ranges are selected based on authors' experience of observing many production lines. Similar ranges are used in [13], [17], and [19]. For each production line, we discrete-event simulate the behavior of the production line. The data from the first 10 000 time slots are discarded to avoid the effect of a transient. The data from the subsequent 200 000 time slots are used to evaluate PR, ST_i, and BL_i.

For each production system, in the order of $i = 1, 2, \ldots, M$, machine m_i is treated as an unknown machine, and $\hat{p}_i$ is obtained

TABLE I
FREQUENCY FOR ESTIMATION ERROR IN BERNOULLI LINE

Metric \ M	5	10	15	20
$\varepsilon_p < 0.05$	90.76%	81.27%	76.93%	74.29%
$\varepsilon_{PR} < 0.05$	100%	100%	100%	100%

by the proposed method. The simulation is done for 10 000 different lines for each $M = 5$, 10, 15, and 20.

The results are shown in Table I. It turns out that $\epsilon_p < 0.05$ for 90.76% of the 10 000 $M = 5$ lines, 81.27% of the $M = 10$ lines, 76.93% for $M = 15$ lines, and 74.29% for $M = 20$ lines, which indicates that the parameter estimation accuracy is high for most cases. For all the cases considered, $\epsilon_{\text{PR}} < 0.05$.

B. Accuracy in Bottleneck Identification

The most common use of the mathematical model of a production line is to identify the bottleneck machine denoted by BNm. The bottleneck machine is defined as the machine with the largest $\partial\text{PR}/\partial p_i$ among all machines in the line. In this subsection, we investigate the BNm identified using $\hat{p}_i$ is identical to BNm identified using p_is by simulating $M = 5$ lines. Two methods are available to identify the bottleneck: sensitivity analysis that evaluates the partial derivatives using the aggregated model and the arrow method that utilizes ST_i and BL_i instead of directly evaluating the partial derivatives. The arrow method is simple and widely used in practice [13].

Let us denote the bottleneck machine resulting from the first method by BNm_{Agg} and that from the latter by BNm_{Arr}. Both methods are not exact. However, in most cases, they give the same result. Since both are not exact, we also identify the bottleneck machine based on discrete-event simulations: numerically evaluating $\partial\text{PR}/\partial p_i$ by perturbing p_i and carrying out simulations. We denote this bottleneck by BNm_{DES} and regard BNm_{DES} as the true bottleneck.

The investigation is done for 5000 randomly generated serial production lines, whose parameters are selected in the range given in (20). For each line, we first find BNm_{DES}, BNm_{Agg}, and BNm_{Arr}. Then, for each line, we treat m_i as an unknown machine in the order of $i = 1, 2, 3, 4,$ and 5, apply the proposed method to estimate p_i, and find $\widehat{\text{BNm}}_{\text{Agg}}$ and $\widehat{\text{BNm}}_{\text{Arr}}$. The hat notation indicates that these bottleneck machines are found from the mathematical model that contains an unknown machine. As a result, for each production line, we have one value of BNm_{DES}, one value of BNm_{Agg}, one value of BNm_{Arr}, five values of $\widehat{\text{BNm}}_{\text{Agg}}$, and five values of $\widehat{\text{BNm}}_{\text{Arr}}$.

The results in Table II show the frequency that $\widehat{\text{BNm}}_{\text{Agg}}$ is equal to the true bottleneck (BNm_{DES}), and also $\widehat{\text{BNm}}_{\text{Arr}}$ is equal to BNm_{DES}. We also show in Table II the frequency of correct BN identification when all machines are known. The BN identification based on indirect estimation is almost as accurate as that of the cases of all machines being known.

TABLE II
ACCURACY OF BNM IDENTIFICATION WITH UNKNOWN BERNOULLI MACHINE

BNm_{DES} is equal to	Frequency(%)	BNm_{DES} is equal to	Frequency(%)
$\widehat{BNm}_{Agg}$	96.85	BNm_{Agg}	96.1
$\widehat{BNm}_{Arr}$	89.6	BNm_{Arr}	91.68

V. INDIRECT ESTIMATION FOR SYNCHRONOUS EXPONENTIAL LINES

A. Analysis of Performance Measures

The exact solutions for PR, ST, and BL exist only for $M = 2$ [13]. They are obtained from a Markov chain model whose states are buffer occupancy and the machine states (up or down). For real numbers μ_1, λ_1, μ_2, and λ_2 and a natural number N_1, define the function

$$Q(\lambda_1, \mu_1, \lambda_2, \mu_2, N_1) = \begin{cases} \dfrac{(1-e_1)(1-\Phi)}{1-\Phi e^{-\beta N_1}}, & \text{if } e_1 \neq e_2, \\ \dfrac{e_1 A}{A + \lambda_2\mu_1\Psi N_1}, & \text{otherwise} \end{cases} \tag{21}$$

where $e_i = \frac{\mu_i}{\mu_i+\lambda_i} (i = 1, 2)$, $\Phi = \frac{e_1(1-e_2)}{e_2(1-e_1)}$, $\Psi = \lambda_1 + \lambda_2 + \mu_1 + \mu_2$, $A = (\lambda_1 + \lambda_2)(\mu_1 + \mu_2)$, and $\beta = \frac{\Psi(\lambda_1\mu_2 - \lambda_2\mu_1)}{A}$. The function $Q(\lambda_1, \mu_1, \lambda_2, \mu_2, N_1)$ represents the conditional probability of the event that the buffer is empty and m_1 is down under the condition that m_2 is down. Then, the performance measures of the two-machine line are given by

$$\begin{aligned} \text{PR} &= e_2[1 - Q(\lambda_1, \mu_1, \lambda_2, \mu_2, N_1)] \\ &= e_1[1 - Q(\lambda_2, \mu_2, \lambda_1, \mu_1, N_1)], \end{aligned} \tag{22}$$

$$\text{BL}_1 = e_1 Q(\lambda_2, \mu_2, \lambda_1, \mu_1, N_1), \tag{23}$$

$$\text{ST}_2 = e_2 Q(\lambda_1, \mu_1, \lambda_2, \mu_2, N_1). \tag{24}$$

For production lines with $M > 2$, the aggregation method [13] is used to obtain the performance measures. Forward and backward parameters, obtained by a recursive procedure, satisfy

$$\mu_1^f = \mu_1, \tag{25}$$

$$\mu_i^f = \mu_i[1 - Q(\lambda_{i-1}^f, \mu_{i-1}^f, \lambda_i^b, \mu_i^b, N_{i-1})],\ i = 2, \ldots, M \tag{26}$$

$$\lambda_1^f = \lambda_1, \tag{27}$$

$$\lambda_i^f = \lambda_i + \mu_i Q(\lambda_{i-1}^f, \mu_{i-1}^f, \lambda_i^b, \mu_i^b, N_{i-1}),\ i = 2, \ldots, M, \tag{28}$$

$$\mu_i^b = \mu_i[1 - Q(\lambda_{i+1}^b, \mu_{i+1}^b, \lambda_i^f, \mu_i^f, N_i)],\ i = 1, \ldots, M-1, \tag{29}$$

$$\mu_M^b = \mu_M, \tag{30}$$

$$\lambda_i^b = \lambda_i + \mu_i Q(\lambda_{i+1}^b, \mu_{i+1}^b, \lambda_i^f, \mu_i^f, N_i),\ i = 1, \ldots, M-1, \tag{31}$$

$$\lambda_M^b = \lambda_M. \tag{32}$$

Performance measures, again denoted with "hat" to indicate that they are approximations, are given by

$$\begin{aligned}\widehat{\text{PR}} &= e_M^f = e_1^b \\ &= e_{i+1}^b[1 - Q(\lambda_i^f, \mu_i^f, \lambda_{i+1}^b, \mu_{i+1}^b, N_i)], \\ &= e_i^f[1 - Q(\lambda_{i+1}^b, \mu_{i+1}^b, \lambda_i^f, \mu_i^f, N_i)],\ \ i = 1, \ldots, M-1,\end{aligned} \tag{33}$$

$$\widehat{\text{BL}}_i = e_i Q(\lambda_{i+1}^b, \mu_{i+1}^b, \lambda_i^f, \mu_i^f, N_i),\ \ i = 1, \ldots, M-1, \tag{34}$$

$$\widehat{\text{ST}}_i = e_i Q(\lambda_{i-1}^f, \mu_{i-1}^f, \lambda_i^b, \mu_i^b, N_{i-1}),\ \ i = 2, \ldots, M. \tag{35}$$

Here, $e_i^f = \frac{\mu_i^f}{\mu_i^f + \lambda_i^f}$ and $e_i^b = \frac{\mu_i^b}{\mu_i^b + \lambda_i^b}$.

B. Two-Machine Lines ($M = 2$)

First, we introduce stationary probabilities of the boundary states defined in [13]

$$\begin{aligned}P_{0,00} &= P\{(b_1 \text{ is empty}) \wedge (m_1 \text{ is down}) \wedge (m_2 \text{ is down})\}, \\ P_{0,11} &= P\{(b_1 \text{ is empty}) \wedge (m_1 \text{ is up}) \wedge (m_2 \text{ is up})\}, \\ P_{N_1,00} &= P\{(b_1 \text{ is full}) \wedge (m_1 \text{ is down}) \wedge (m_2 \text{ is down})\}, \\ P_{N_1,11} &= P\{(b_1 \text{ is full}) \wedge (m_1 \text{ is up}) \wedge (m_2 \text{ is up})\}\end{aligned}$$

where $\wedge$ denotes the "and" condition for the events involved. Note that the eight events associated with the above four boundary probabilities are mutually exclusive.

Theorem 3: Consider the production lines shown in Fig. 1 with $M = 2$. Let m_i be the unknown machine.

1) If $i = 1$, then the exponential parameters λ_1 and μ_1 are given by

$$\mu_1 = \frac{\text{ST}_2}{P_{0,00}}\lambda_2 - \mu_2, \tag{36}$$

$$\lambda_1 = \frac{\text{ST}_2}{P_{0,11}}\mu_1 - \lambda_2. \tag{37}$$

2) If $i = 2$, then the exponential parameters λ_2 and μ_2 are given by

$$\mu_2 = \frac{\text{BL}_1}{P_{N_1,00}}\lambda_1 - \mu_1, \tag{38}$$

$$\lambda_2 = \frac{\text{BL}_1}{P_{N_1,11}}\mu_2 - \lambda_1. \tag{39}$$

Proof: See Appendix ■

Here, $P_{0,00}$ and $P_{0,11}$ can be obtained without the operational information of m_1. Since the exponential line is analyzed based on the flow (continuous) model, the probability for the event $(b_1 \text{ is empty}) \wedge (m_1 \text{ is up}) \wedge (m_2 \text{ is down})$ is zero. The state of each machine is determined independently from the other machine; thus, we obtain

$$P_{0,00} = P\{(b_1 \text{ is empty}) \wedge (m_2 \text{ is down})\} \tag{40}$$

and ST is the probability of the event $(b_1 \text{ is empty}) \wedge (m_1 \text{ is down}) \wedge (m_2 \text{ is up})$ and, thus,

$$P_{0,11} = P\{(b_1 \text{ is empty}) \wedge (m_2 \text{ is up})\} - \text{ST}_2. \tag{41}$$

Both of $P_{0,00}$ and $P_{0,11}$ are obtained from the logs of m_2.

Similarly, $P_{N_1,00}$ and $P_{N_1,11}$ can be obtained from the log of m_1 as follows:

$$P_{N_1,00} = P\{(b_1 \text{ is full}) \wedge (m_1 \text{ is down})\}, \tag{42}$$

$$P_{N_1,11} = P\{(b_1 \text{ is full}) \wedge (m_1 \text{ is up})\} - \text{BL}_1. \tag{43}$$

According to the result 1), the reliability parameters of the unknown machine, i.e., λ_1 and μ_1, can be obtained indirectly [not using (2)] only using the log of m_2. The parameters λ_2 and μ_2 are obtained from the log of m_2 directly using (2). The result 2) applies for case when m_2 is unknown.

C. Lines With More Than Two Machines ($M > 2$)

When $M > 2$, results of the aggregation method, i.e., (25)–(32), (34), and (35), are used for the indirect estimation of an unknown machine. Four boundary probabilities are defined for the aggregated, virtual two-machine line shown in Fig. 2

$$\begin{aligned}P_{0,00}^{\text{agg},i} &= P\{(b_i \text{ is empty}) \wedge (m_i^f \text{ is down}) \wedge (m_{i+1}^b \text{ is down})\}, \\ P_{0,11}^{\text{agg},i} &= P\{(b_i \text{ is empty}) \wedge (m_i^f \text{ is up}) \wedge (m_{i+1}^b \text{ is up})\}, \\ P_{N_i,00}^{\text{agg},i} &= P\{(b_i \text{ is full}) \wedge (m_i^f \text{ is down}) \wedge (m_{i+1}^b \text{ is down})\}, \\ P_{N_i,11}^{\text{agg},i} &= P\{(b_i \text{ is full}) \wedge (m_i^f \text{ is up}) \wedge (m_{i+1}^b \text{ is up})\}\end{aligned}$$

where $i = 1, 2, \ldots, M-1$.

Theorem 4: Consider the production lines shown in Fig. 1 with $M > 2$. Let m_i be an unknown machine.

1) If $i \in \{2, \ldots, M-1\}$, then the parameters $\hat{\mu}_i$ and $\hat{\lambda}_i$ are given by

$$\hat{\mu}_i = \frac{\mu_i^f}{1 - Q(\lambda_{i-1}^f, \mu_{i-1}^f, \lambda_i^b, \mu_i^b, N_{i-1})}, \tag{44}$$

$$\hat{\lambda}_i = \lambda_i^f - \hat{\mu}_i Q(\lambda_{i-1}^f, \mu_{i-1}^f, \lambda_i^b, \mu_i^b, N_{i-1}) \tag{45}$$

where μ_i^f, λ_i^f, μ_i^b, and λ_i^b are

$$\mu_i^f = \frac{\text{ST}_{i+1}(\frac{e_{i+1}^b}{e_{i+1}})}{P_{0,00}^{\text{agg},i}}\lambda_{i+1}^b - \mu_{i+1}^b, \tag{46}$$

$$\lambda_i^f = \frac{\text{ST}_{i+1}(\frac{e_{i+1}^b}{e_{i+1}})}{P_{0,11}^{\text{agg},i}}\mu_i^f - \lambda_{i+1}^b, \tag{47}$$

$$\mu_i^b = \frac{\text{BL}_{i-1}(\frac{e_{i-1}^f}{e_{i-1}})}{P_{N_{i-1},00}^{\text{agg},i-1}}\lambda_{i-1}^f - \mu_{i-1}^f, \tag{48}$$

$$\lambda_i^b = \frac{\text{BL}_{i-1}(\frac{e_{i-1}^f}{e_{i-1}})}{P_{N_{i-1},11}^{\text{agg},i-1}}\mu_i^b - \lambda_{i-1}^f \tag{49}$$

and μ^f_{i-1}, λ^f_{i-1}, μ^b_{i-1}, and λ^b_{i-1} are given by

$$\mu^f_{i-1} = \mu_{i-1}\left(1 - \frac{\mathrm{ST}_{i-1}}{e_{i-1}}\right), \tag{50}$$

$$\lambda^f_{i-1} = \lambda_{i-1} + \mu_{i-1}\left(\frac{\mathrm{ST}_{i-1}}{e_{i-1}}\right), \tag{51}$$

$$\mu^b_{i+1} = \mu_{i+1}\left(1 - \frac{\mathrm{BL}_{i+1}}{e_{i+1}}\right), \tag{52}$$

$$\lambda^b_{i+1} = \lambda_{i+1} + \mu_{i+1}\left(\frac{\mathrm{BL}_{i+1}}{e_{i+1}}\right). \tag{53}$$

Alternatively, $\hat{\lambda}_i$ and $\hat{\mu}_i$ can also be obtained by

$$\hat{\mu}_i = \frac{\mu^b_i}{1 - Q(\lambda^b_{i+1}, \mu^b_{i+1}, \lambda^f_i, \mu^f_i, N_i)}, \tag{54}$$

$$\hat{\lambda}_i = \lambda^b_i - \hat{\mu}_i Q(\lambda^b_{i+1}, \mu^b_{i+1}, \lambda^f_i, \mu^f_i, N_i). \tag{55}$$

2) If $i = 1$, then the exponential parameters $\hat{\mu}_1$ and $\hat{\lambda}_1$ are given by

$$\hat{\mu}_1 = \frac{\mathrm{ST}_2(\frac{e^b_2}{e_2})}{P^{\mathrm{agg},1}_{0,00}}\lambda^b_2 - \mu^b_2, \tag{56}$$

$$\hat{\lambda}_1 = \frac{\mathrm{ST}_2(\frac{e^b_2}{e_2})}{P^{\mathrm{agg},1}_{0,11}}\hat{\mu}_1 - \lambda^b_2 \tag{57}$$

where

$$\mu^b_2 = \mu_2\left(1 - \frac{\mathrm{BL}_2}{e_2}\right),$$

$$\lambda^b_2 = \lambda_2 + \mu_2\left(\frac{\mathrm{BL}_2}{e_2}\right).$$

3) If $i = M$, then the exponential parameters $\hat{\mu}_M$ and $\hat{\lambda}_M$ are given by

$$\hat{\mu}_M = \frac{\mathrm{BL}_{M-1}(\frac{e^f_{M-1}}{e_{M-1}})}{P^{\mathrm{agg},M-1}_{N_{M-1},00}}\lambda^f_{M-1} - \mu^f_{M-1}, \tag{58}$$

$$\hat{\lambda}_M = \frac{\mathrm{BL}_{M-1}(\frac{e^f_{M-1}}{e_{M-1}})}{P^{\mathrm{agg},M-1}_{N_{M-1},11}}\hat{\mu}_M - \lambda^f_{M-1} \tag{59}$$

where

$$\mu^f_{M-1} = \mu_{M-1}\left(1 - \frac{\mathrm{ST}_{M-1}}{e_{M-1}}\right),$$

$$\lambda^f_{M-1} = \lambda_{M-1} + \mu_{M-1}\left(\frac{\mathrm{ST}_{M-1}}{e_{M-1}}\right).$$

Proof: See Appendix. ∎

Note that we obtain ST_{i-1}, BL_{i-1}, μ_{i-1}, and λ_{i-1} from the log of m_{i-1}, and ST_{i+1}, BL_{i+1} μ_{i+1}, and λ_{i+1} from the log of m_{i+1}. Thus, the parameters μ^f_{i-1}, λ^f_{i-1}, μ^b_{i+1}, and λ^b_{i+1} in (50)–(53) are obtained from the logs of m_{i-1} and m_{i+1}.

However, obtaining μ^f_i, λ^f_i μ^b_i, and λ^b_i from (46) to (49) is not feasible because $P^{\mathrm{agg},i}_{0,00}$, $P^{\mathrm{agg},i}_{0,11}$, $P^{\mathrm{agg},i-1}_{N_{i-1},00}$, and $P^{\mathrm{agg},i-1}_{N_{i-1},11}$ are quantities for the virtual, aggregated machines. A measurement-based alternative is necessary to proceed further. We first consider μ^f_i in (46). The only unknown in (46) is $P^{\mathrm{agg},i}_{0,00}$. However, the approach of directly approximating $P^{\mathrm{agg},i}_{0,00}$ by available measurement is in vain. Instead, we proceed as follows. We approximate μ^f_i using ST_{i+1} and $P^i_{0,00}$, which are obtained from the log of m_{i+1}. The values of ST_{i+1} and $P^i_{0,00}$ well approximate the probabilities that are defined in the virtual line consisting of m^f_i, b_i, and m_{i+1}. Thus, using 1) (36) of Theorem 3 yields the following approximation:

$$\mu^f_i \approx \frac{\mathrm{ST}_{i+1}}{P^i_{0,00}}\lambda_{i+1} - \mu_{i+1}. \tag{60}$$

We now explain why ST_{i+1} and $P^i_{0,00}$ approximate well the probabilities defined in the virtual line. The probability of ST_{i+1} can be written out as

$$\mathrm{ST}_{i+1} = P\Big\{(b_i \text{ empty}) \wedge \{(m_i \text{ down}) \vee (m_i \text{ starved})\} \wedge \{(m_{i+1} \text{ up}) \wedge (m_{i+1} \text{ not blocked})\}\Big\}$$

where $\vee$ denotes the "or" condition. The event $(m_i \text{ down}) \vee (m_i \text{ starved})$ implies $(m^f_i \text{ down})$: One can see this in the equality $(1 - e_i) + \widehat{\mathrm{ST}}_i = (1 - e^f_i)$ obtained from (26) and (28). Further, the event $(b_i \text{ empty}) \wedge \{(m_i \text{ down}) \vee (m_i \text{ starved})\} \wedge (m_{i+1} \text{ blocked})$ is null because it implies m_{i+1} blocked and starved simultaneously, which is not possible in the flow model of production lines. Thus, we can rewrite ST_{i+1} as

$$\mathrm{ST}_{i+1} \approx P\left\{(b_i \text{ empty}) \wedge (m^f_i \text{ down}) \wedge (m_{i+1} \text{ up})\right\}.$$

Clearly, this supports ST_{i+1} can be considered by the probability in line consisted of m^f_i, b_i, and m_{i+1}.

The probability $P^i_{0,00}$ in (60) is defined as

$$P^i_{0,00} = P\Big\{(b_i \text{ empty}) \wedge \{(m_i \text{ down}) \vee (m_i \text{ starved})\} \wedge \{(m_{i+1} \text{ down}) \vee (m_{i+1} \text{ blocked})\}\Big\}.$$

But using similar argument as for ST_{i+1}, it reduces to

$$P^i_{0,00} \approx P\left\{(b_i \text{ empty}) \wedge (m^f_i \text{ down}) \wedge (m_{i+1} \text{down})\right\}.$$

Thus, ST_{i+1} and $P^i_{0,00}$ are the probabilities measured in the actual production line with $M > 2$, but approximate the probabilities that are defined in the virtual two-machine line consisting of m^f_i and m_{i+1}. This justifies (60). Finally, we can calculate ST_{i+1} from the log of m_{i+1}. We can also calculate $P^i_{0,00}$ from the log of m_{i+1} by measuring the event $(b_i \text{ empty}) \wedge (m_{i+1} \text{ down})$.

Next, we derive an approximation for λ^f_i in (47). Contrary to $P^{\mathrm{agg},i}_{0,00}$, a direct evaluation of $P^{\mathrm{agg},i}_{0,11}$ from the log of m_{i+1} is possible. We approximate $P^{\mathrm{agg},i}_{0,11}$ by $P^i_{0,11}$ defined as

$$P^i_{0,11} = P\Big\{(b_i \text{ empty}) \wedge \{(m_i \text{ up}) \wedge (m_i \text{ not starved})\} \wedge$$

$$\{(m_{i+1} \text{ up}) \wedge (m_{i+1} \text{ not blocked})\}\Big\}.$$

Note that the event $(m_{i+1} \text{ blocked})$ and the event $(b_i \text{ empty}) \wedge (m_i \text{ up}) \wedge (m_i \text{ not starved})$ may not simultaneously occur. Thus

$$P^i_{0,11} \approx P\left\{(b_i \text{ empty}) \wedge (m^f_i \text{ up}) \wedge (m^b_{i+1} \text{ up})\right\} = P^{\text{agg},i}_{0,11}. \quad (61)$$

The forward aggregation parameter λ^f_i is obtained by (47) where $P^{\text{agg},i}_{0,11}$ is replaced by $P^i_{0,11}$. The value of $P^i_{0,11}$ is obtained from the log of m_{i+1} in the following manner:

$$P^i_{0,11} = P\{(b_i \text{ empty}) \wedge (m_{i+1} \text{ up}) \wedge (m_{i+1} \text{ not blocked})\} - \text{ST}_{i+1}.$$

This is because the event $(b_i \text{ empty}) \wedge \{(m_{i+1} \text{ up}) \wedge (m_{i+1} \text{ not blocked})$ consists of two exclusive events $(m_i \text{ up}) \wedge (m_i \text{ not starved})$ and $(m_i \text{ down}) \vee (m_i \text{ starved})$. The probability of the first of the two is $P^i_{0,11}$ and that of the second is ST_{i+1}.

Similarly, we have

$$\mu^b_i \approx \frac{\text{BL}_{i-1}}{P^{i-1}_{N_{i-1},00}}\lambda_{i-1} - \mu_{i-1} \quad (62)$$

where

$$P^{i-1}_{N_{i-1},00} = P\Big\{(b_{i-1} \text{ full}) \wedge \{(m_{i-1} \text{ down}) \vee (m_{i-1} \text{ starved})\} \wedge \{(m_i \text{ down}) \vee (m_i \text{ blocked})\}\Big\}$$

and λ^b_i is from (49) with

$$P^{\text{agg},i-1}_{N_{i-1},11} \approx P^{i-1}_{N_{i-1},11} \quad (63)$$

where

$$P^{i-1}_{N_{i-1},11} = P\Big\{(b_{i-1} \text{ full}) \wedge \{(m_{i-1} \text{ up}) \wedge (m_{i-1} \text{ not starved})\} \wedge \{(m_i \text{ up}) \wedge (m_i \text{ not blocked})\}\Big\}.$$

With the approximations (60)–(63), all the forward and backward parameters μ^f_i, λ^f_i, μ^b_i, and λ^b_i are obtained from the logs of both sides of the unknown machines m_{i-1} and m_{i+1} $(i = 2, 3, \ldots, M-1)$. Items 2) and 3) of Theorem 4 are also obtained using (60)–(63).

VI. Accuracy of Estimation on Synchronous Exponential Lines

Discrete event simulations are carried out using 500 randomly generated production lines, respectively, for each one of $M = 5$, 10, 15, and 20. The total of 2000 different production lines are considered. Machine parameters and buffer capacities are randomly generated by $e_i \in [0.75, 0.95]$, $\mu_i \in [0.2, 0.4]$, $\lambda_i = \frac{1-e_i}{e_i}\mu_i$, and $N_i \in [2, 5]$. The data from the first 5000 downs are discarded to avoid the effect of transient. The data from the subsequent 50 000 downs are used to evaluate ST_i, BL_i, $P^i_{0,00}$, $P^i_{0,11}$, $P^i_{N_i,00}$, $P^i_{N_i,11}$, and PR.

TABLE III
Frequency for Estimation Error in Synchronous Exponential Line

Metric \ M	5	10	15	20
$\varepsilon_{\mu_i} < 0.05$	58%	30.74%	25.91%	27.61%
$\varepsilon_{\lambda_i} < 0.05$	12.44%	7.16%	7.04%	10.4%
$\varepsilon_{e_i} < 0.05$	69.56%	50.08%	44.88%	43.25%
$\varepsilon_{PR_i} < 0.05$	98.36%	93.98%	84.69%	71.02%
$E[\varepsilon_{\mu_i}]$	0.0823	0.1406	0.1580	0.1610
$E[\varepsilon_{\lambda_i}]$	0.4143	0.6705	0.7606	0.8062
$E[\varepsilon_{e_i}]$	0.0443	0.0974	0.1036	0.1122
$E[\varepsilon_{PR_i}]$	0.0168	0.0276	0.0389	0.0441

Three metrics for the accuracy are defined by

$$\epsilon_\lambda = \frac{|\lambda_i - \hat{\lambda}_i|}{\lambda_i},\ \epsilon_\mu = \frac{|\mu_i - \hat{\mu}_i|}{\mu_i},\ \epsilon_e = \frac{|e_i - \hat{e}_i|}{e_i} \quad (64)$$

where $\hat{e}_i = \hat{\mu}_i/(\hat{\lambda}_i + \hat{\mu}_i)$. The total of 25 000 values [25 000 = 500 lines times (5+10+15+20 machines per lines)] are obtained for each of the three metrics, respectively, from the simulations. Additionally, we define

$$\epsilon_{\text{PR}} = \frac{|\text{PR}_s - \text{PR}_i|}{\text{PR}_s} \quad (65)$$

where PR_s is the production rate obtained by the simulation, and PR_i is the production rate obtained by aggregating the line with m_i being unknown.

The results are shown in Table III. It turns out that the average of 25 000 values of ϵ_λ is 0.7376, the average of ϵ_μ is 0.151, that of ϵ_e is 0.0996, and that of ϵ_{PR} is 0.0365. All except the first exhibits accuracy less than about 15%. However, the accuracy for λ is poor. This might be because the sensitivity of λ on PR is much lower than that of μ as long as e is reasonably high ($e_i \in [0.75, 0.95]$). For ϵ_{PR}, in addition to the average, its distribution is looked at. It turns out that $\epsilon_{\text{PR}} < 0.05$ for 98.36%

TABLE IV
ACCURACY OF BNM IDENTIFICATION WITH UNKNOWN EXPONENTIAL MACHINE

BNm_{DES} is equal to	Frequency(%)	BNm_{DES} is equal to	Frequency(%)
$\widehat{BNm}_{Agg}$	82.4	BNm_{Agg}	85.4

of the $M = 5$ cases, 93.98% for $M = 10$, 84.69% for $M = 15$, and 71.02% for $M = 20$, which also indicates that the models with estimated parameters are well validated.

Next, we analyze the accuracy of BNm identification. The BNm is defined as the machine that has the largest value of $\partial \mathrm{PR}/\partial \mu_i$. We discrete-event simulate 500 different lines with $M = 5$ to evaluate the accuracy of BNm identification when a machine is unknown. The BNm is identified by numerically evaluating the partial derivative. The results are presented in Table IV. The BNm identification using the indirect estimation is as accurate as the case in which all machines are known. It is only 3% less than the known case.

VII. CONCLUSION

We developed a data-driven method that indirectly estimates the unknown machine's reliability parameter from the data of neighboring machines. Reliability characteristics considered in this work are the Bernoulli reliability model for discrete systems and the exponential reliability for continuous systems. As future work, methods of autonomously detecting unknown machines may be considered as other ingredients to realize smart production systems. Then, data-driven autonomous update of the production system model, along with the indirect estimation method developed in this work, becomes closer to the reality. Obviously, the results of this work may contribute to increase the resiliency of the smart production systems for the situation when automated logging devices of some machines fail.

APPENDIX

Proof of Theorem 1: We prove that (11) has only one solution in the range $0 < x < 1$. The function $Q(x, y, N_1)$ is continuous and strictly decreasing in x and takes values on (0,1) for any $0 < y < 1$ and positive integer N_1[13]. Also, it is straightforward to show that $Q(x, y, N_1)$ satisfies

$$\lim_{x\to 0} Q(x, y, N) = 1, \text{ and } \lim_{x\to 1} Q(x, y, N) = 0$$

for any $0 < y < 1$ and positive integer N_1. The inequality $0 < \mathrm{ST}_2 < p_2$ is always satisfied by the definition of ST_2; thus, $0 < \mathrm{ST}_2/P_2 < 1$. Therefore, there is only one valid solution that satisfies (11).

In the case that m_2 is an unknown machine, the proof is also in the same manner. ■

Proof of Theorem 2: Equation (13) comes from (6). Unknowns in (13) are p_i^b, p_i^f, and p_{i+1}^b. To obtain p_i^b, Theorem 1 is adopted to virtual two-machine line with m_{i-1}^f, m_i^b, and b_{i-1}: p_i^b is the unique solutions of the following equations in x

$$Q(x, p_{i-1}^f, N_{i-1}) = \frac{\mathrm{BL}_{i-1}^f}{p_{i-1}^f} \tag{66}$$

where BL_{i-1}^f is the blockage probability of m_{i-1}^f. p_{i-1}^f can be calculated from (5) and (6). The right side of (66) can be changed as shown in (15) using (4) and (9).

Similar argument is applied to obtain p_i^f with virtual two-machine line with m_i^f, m_{i+1}^b, and b_i.

When $i = M$, the boundary condition of recursive aggregation procedure $p_M^b = p_M$ is used. Thus, solving (66) in x leads to indirect estimation of p_M. The indirect estimation of p_1^f can be similar to the boundary condition $p_1^f = p_1$. ■

Proof of Theorem 3: The following relationship is defined in [13]:

$$\frac{\mathrm{ST}_2}{P_{0,00}} = \frac{\mu_1+\mu_2}{\lambda_2}, \quad \frac{\mathrm{ST}_2}{P_{0,11}} = \frac{\lambda_1+\lambda_2}{\mu_1},$$
$$\frac{\mathrm{BL}_1}{P_{N,00}} = \frac{\mu_1+\mu_2}{\lambda_1}, \quad \frac{\mathrm{BL}_1}{P_{N,11}} = \frac{\lambda_1+\lambda_2}{\mu_2}.$$

With these relationships, we can derive Theorem 3. ■

Proof of Theorem 4: We can obtain (44), (45), (54), and (55) from (26), (28), (29), and (31).

Consider the production line with two virtual machines m_i^f and m_{i+1}^b. Theorem 3 is used to derive the aggregation parameters as follows:

$$\mu_i^f = \frac{\mathrm{ST}_{i+1}^{\mathrm{agg}}}{P_{0,00}^{\mathrm{agg},i}} \lambda_{i+1}^b - \mu_{i+1}^b \tag{67}$$

$$\lambda_i^f = \frac{\mathrm{ST}_{i+1}^{\mathrm{agg}}}{P_{0,11}^{\mathrm{agg},i}} \mu_i^f - \lambda_{i+1}^b \tag{68}$$

where $i = 2, 3, \ldots, M-1$, and $\mathrm{ST}_{i+1}^{\mathrm{agg}}$ is the starvation probability of m_{i+1}^b defined as

$$\mathrm{ST}_{i+1}^{\mathrm{agg}} = e_{i+1}^b Q(\lambda_i^f, \mu_i^f, \lambda_{i+1}^b, \mu_{i+1}^b, N_i) \tag{69}$$

by the definition of starvation (24) for $M = 2$ line.

We can calculate $Q(\lambda_i^f, \mu_i^f, \lambda_{i+1}^b, \mu_{i+1}^b, N_i)$ using (35) as follows:

$$Q(\lambda_i^f, \mu_i^f, \lambda_{i+1}^b, \mu_{i+1}^b, N_i) = \frac{\widehat{\mathrm{ST}}_{i+1}}{e_{i+1}}. \tag{70}$$

Thus, $\mathrm{ST}_{i+1}^{\mathrm{agg}}$ can be replaced with $\frac{e_{i+1}^b}{e_{i+1}} \mathrm{ST}_{i+1}$.

Similar explanation for m_i^f can be used to derive backward parameters (48) and (49), and the first and the last machine's parameters (56)–(59).

With (70) and the results of the aggregation procedure (26)–(31), (50)–(53) can be obtained. ■

REFERENCES

[1] A. Sanders, C. Elangeswaran, and J. Wulfsberg, "Industry 4.0 implies lean manufacturing: Research activities in industry 4.0 function as enablers for lean manufacturing," *J. Ind. Eng. Manage.*, vol. 9, no. 3, pp. 811–833, 2016.

[2] Y. Wang and L. Li, "Time-of-use based electricity demand response for sustainable manufacturing systems," *Energy*, vol. 63, no. 15, pp. 233–244, Dec. 2013.

[3] J. Wan, M. Yi, D. Li, C. Zhang, S. Wang, and K. Zhou, "Mobile services for customization manufacturing systems: An example of industry 4.0," *IEEE Access*, vol. 4, pp. 8977–8986, 2016.

[4] Y. Zhang, Z. Guo, J. Lv, and Y. Liu, "A framework for smart production-logistics systems based on CPS and industrial IoT," *IEEE Trans. Ind. Informat.*, vol. 14, no. 9, pp. 4019–4032, Sep. 2018.

[5] D. Hästbacka *et al.*, "Dynamic edge and cloud service integration for industrial IoT and production monitoring applications of industrial cyber-physical systems," *IEEE Trans. Ind. Informat.*, vol. 18 no. 1, pp. 498–508, Jan. 2022.

[6] E. Lima, R. Bayot, P. Brito, N. Rodrigues, B. Ribeiro, and N. Lopes, "Business analytical framework for the manufacturing industry," in *Proc. IEEE 19th Int. Conf. Ind. Informat.*, 2021, pp. 1–8.

[7] B. Otto *et al.*, "Reference architecture model for the industrial data space," Fraunhofer, Munich, Germany, Tech. Rep., 2017, doi: 10.13140/RG.2.2.17352.11529.

[8] Á. A. Alonso, J. M. Pozo, F. Cantera, De la Vega, and J. J. Hierro, "Industrial data space architecture implementation using FIWARE," *Sensors*, vol. 18, no. 7, 2018, Art. no. 2226.

[9] M. T. Telsang, *Industrial Engineering and Production Management*. New Delhi, India: S. Chand, 2006.

[10] A. Bonci, M. Pirani, and S. Longhi, "Tiny cyber-physical systems for performance improvement in the factory of the future," *IEEE Trans. Ind. Informat.*, vol. 15, no. 3, pp. 1598–1608, Mar. 2019.

[11] P. Alavian, Y. Eun, S. M. Meerkov, and L. Zhang, "Smart production systems: Automating decision-making in manufacturing environment," *Int. J. Prod. Res.*, vol. 58, no. 3, pp. 828–845, 2020.

[12] Z. Ye, Z. Cai, S. Si, S. Zhang, and H. Yang, "Competing failure modeling for performance analysis of automated manufacturing systems with serial structures and imperfect quality inspection," *IEEE Trans. Ind. Informat.*, vol. 16, no. 10, pp. 6476–6486, Oct. 2020.

[13] J. Li and S. M. Meerkov, *Production Systems Engineering*. Cham, Switzerland: Springer, 2008.

[14] P. Alavian, Y. Eun, K. Liu, S. M. Meerkov, and L. Zhang, "The (α, β)-precise estimates of MTBF and MTTR: Definition, calculation, and observation time," *IEEE Trans. Automat. Sci. Eng.*, vol. 18, no. 3, pp. 1469–1477, Jul. 2021.

[15] Y. Sun, T. Zhu, L. Zhang, and P. Denno, "Parameter identification for bernoulli serial production line model," *IEEE Trans. Automat. Sci. Eng.*, vol. 18, no. 4, pp. 2115–2127, Oct. 2021.

[16] Y. Won, S. Kim, K.-J. Park, and Y. Eun, "Continuous productivity improvement using IoE data for fault monitoring: An automotive parts production line case study," *Sensors*, vol. 21, no. 21, 2021, Art. no. 7366.

[17] J. Arinez, S. Biller, S. M. Meerkov, and L. Zhang, "Quality/Quantity improvement in an automotive paint shop: A case study," *IEEE Trans. Automat. Sci. Eng.*, vol. 7, no, 4, pp. 755–761, Oct. 2010.

[18] J. Li, "Continuous improvement at toyota manufacturing plant: Applications of production systems engineering methods," *Int. J. Prod. Res.*, vol. 51, no. 23/24, pp. 7235–7249, 2013.

[19] Z. Jia, L. Zhang, J. Arinez, and G. Xiao, "Finite production run-based serial lines with Bernoulli machines: Performance analysis, bottleneck, and case study," *IEEE Trans. Automat. Sci. Eng.*, vol. 13, no. 1, pp. 134–148, Jan. 2016.

[20] X. Xie and J. Li, "Modeling analysis and continuous improvement of food production systems: A case study at a meat shaving and packaging line," *J. Food Eng.*, vol. 113, no. 2, pp. 344–350, Nov. 2012.

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