IC566: Random Signals and Random Processes (Spring 2019)

Assignment 2 (Due: April 8, 2019)

1. Using a biased coin to make an unbiased decision.

Alice and Bob want to choose between the opera and the movies by tossing a fair coin.

Unfortunately, the only available coin is biased (though the bias is not known exactly). How can they use the biased coin to make a decision so that either option (opera or the movies) is equally likely to be chosen?

2. St.Petersburg paradox. You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

3. Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red, independently of the others.

(a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?

(b) Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?

4. Let a random variable X denote the number of independent coin tosses required until you get the first head. The probability of a head per coin toss is given as p.

(a) What is the PMF px(k) of X?

(b) What is the PMF $p_{X-1|X>1}(k)$ of X-1 given that X>1?

(c) Calculate E[X].

5. You drive 300 km from Daegu to Seoul. Your speed denoted by a random variable V is uniformly distributed between 60 and 100 km/h. The duration of the trip is denoted by a random variable T.

(a) What is the CDF $F_T(t)$ of T?

(b) What is the PDF $f_T(t)$ of T?

6. Consider an experiment of tossing two coins three times. Coin A is fair, but coin B is not, with P(head) = 1/4 and P(tail) = 3/4. Consider a pair of random variables (X, Y), where X denotes the number of heads resulting from coin A and Y denotes the number of heads resulting from coin B.

(a) Find the range of (X, Y).

(b) Find the joint PMF of (X, Y).

(c) Find $P(X=Y)$, $P(X<Y)$, and $P(X+Y\leq 3)$.

7. Let $Y = e^x$ denote a function of a random variable X.

(a) Find the formula of the CDF of Y if $X \sim N(\mu, \sigma^2)$. You may leave an integral in the answer.

(b) Find the formula of the PDF of Y if $X \sim N(\mu, \sigma^2)$.

(c) Find E(Y) if X is a uniform random variable over (0, 1).

8. Let X and Y be independent uniform random variables over (0, 1). Find the formula of the PDF of $Z = X + Y$.

9. Let X and Y be defined by

$$X = \cos \Theta \qquad Y = \sin \Theta$$

where Θ is a random variable uniformly distributed over $(0, 2\pi)$.

(a) Show that X and Y are uncorrelated.

(b) Show that X and Y are not independent.

10. You break a 1-meter stick into two separate pieces at one random point that is uniformly distributed over the length of the stick. .

(a) Find the mean length of the larger of the two pieces.

(b) What is the probability that the two pieces have the exactly same length?

11. The PDF of a random variable X is given by

$$f_X(x) = \begin{cases} 1/3, & for \;\; 0 < x \leq 1 \\ k, & for \;\; 1 < x \leq 2 \\ 0, & otheriwise \end{cases}$$

where k is a constant.

(a) Find the value of k.

(b) A new random variable Y is defined as

$$\mathrm{Y} = \begin{cases} 1, & for \;\; 0 < x \leq 1 \\ 2, & for \;\; 1 < x \leq 2 \\ 0, & otheriwise \end{cases}$$

Are X and Y independent?

(c) Find the conditional PDFs $f_{X|Y}$ (x|1) and $f_{X|Y}$ (x|2).

(d) Find E[X|Y=1] and E[X|Y=2].

(e) Find E[X].