IC566: Random Signals and Random Processes (Spring 2018)

Assignment 3 (Due: May 14, 2018)

1. Let X be an exponential random variable with parameter λ. Find the mean and variance of X in terms of λ.

2. Patients arrive at the doctor's office according to a Poisson process with rate $\lambda = 1/10$ per minute. The doctor will not see a patient until at least three patients are in the waiting room.

(a) Find the expected waiting time until the first patient is admitted to see the doctor.

(b) What is the probability that nobody is admitted to see the doctor in the first hour?

3. The two teams of the FC-ICE and the Inter-HOT play a mini soccer match. The goals from each team are modeled as a Poisson process, independent of each other. The FC-ICE is assumed to score 2 goals/30 minutes on average while the Inter-HOT 1 goal/30 minutes. This game has regulation times of the first and second half, 30 minutes each.

This is the second match of their home-and-away tournament. The first match was scoreless. Therefore, if they are still scoreless during the regulation, they play the extra time of 30 minutes. If still scoreless after the extra time, they play a shoot-out.

You may use $P(k, t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$ for the probability that a Poisson process has k arrivals for the duration of t or $P(T_1 < t) = 1 - e^{-\lambda t}$ $(t > 0)$ for the probability that the inter-arrival time for the Poisson process is less than t. Of course, you should specify your λ clearly.

(a) What is the probability that both teams play extra time?

(b) What is the probability that both teams play extra time but no shoot out?

(c) What is the expected number of goals from the FC-ICE? You do not count a shoot-out goal after extra time.

(d) What is the expected total time of this match? Again, you do not count shoot-out time.

4. A superstar goalie sometimes mishandles a harmless shot and allows a goal by mistake. The occurrence of these mistakes is assumed to be a Poisson process with $\lambda = 0.02$ mistake/90 minutes. One soccer game takes 90 minutes.

You may use $P(k, t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$ for the probability that a Poisson process has k arrivals for the duration of t or $P(T_1 < t) = 1 - e^{-\lambda t}$ $(t > 0)$ for the probability that the inter-arrival time for the Poisson process is less than t.

(a) What is the probability that the goalie makes one mistake during one soccer game?

(b) This goalie has a plan to play soccer games until he makes the second mistake. Assuming no overtime/shoot-outs and no other issues, what is the expected number of soccer games he would play in his career?

5. A lazy professor issues a homework according to a Poisson process with $\lambda = 4$ homeworks/ semester. One semester is assumed to be 100 days.

You may use $P(k, t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$ for the probability that a Poisson process has k arrivals for the duration of t or $P(T_1 < t) =$

$1 - e^{-\lambda t}$ (t > 0) for the probability that the inter-arrival time for the Poisson process is less than t. You may leave the exponential in your answer.

(a) What is the probability that the class is homework-free during one semester?
(b) The professor had issued the first homework on March 2. What is the probability that the professor (who is totally out-of-mind) issued the second homework before March 9?
(c) Now the professor (who continues to be out-of-mind) flips two fair coins every time he makes a homework. If both coins show heads, he issues a computer-simulation project. What is the probability that this class has at least one project during one semester?