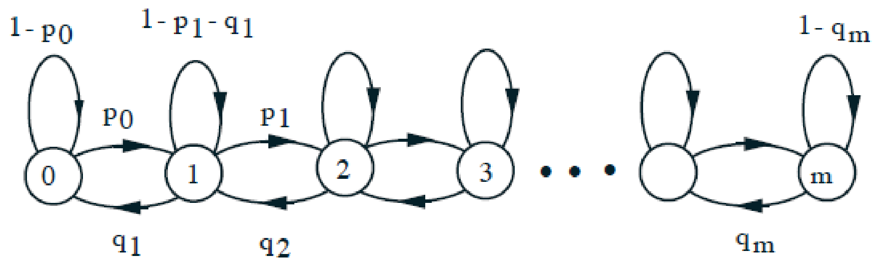
IC566: Random Signals and Random Processes (Spring 2018)

Assignment 4 (Due: May 31, 2018)

1. A birth-death process is shown below with m+1 states with transition probability p_i from state i to i+1, and q_i from i to i-1.



(a) Explain that this process has steady-state probabilities π_i. (i.e., the n-step transition probabilities converge to π_i.**)**

(b) Assume $p_i = p$ and $q_i = q$ (> p) for every i. Also assume that m goes to infinity. Find π_i.

2. Consider a Markov process with two states and transition probability matrix

$$P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

(a) Draw a Markov chain showing two states and the transition probabilities.

(b) Find the stationary distribution π_1 and π_2 of the chain.

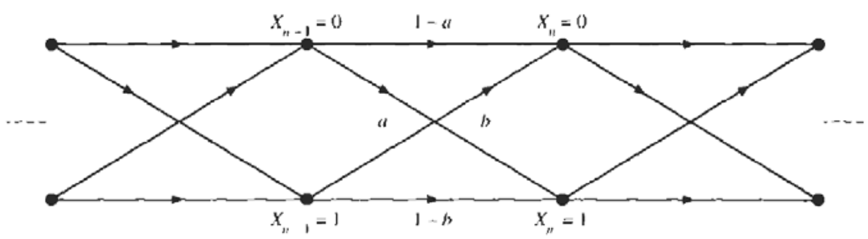
3. An example of a two-state Markov chain is provided by a communication network consisting of the sequence (or cascade) of stages of binary communication channels shown in the figure below. Here X_n denotes the digit leaving the nth stage of the channel and X_0 denotes the digit entering the first stage. The transition probability matrix of this communication network is often called the channel matrix, which is

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \qquad 0 < a < 1, 0 < b < 1$$

Assume that a = 0.1 and b = 0.2, and the initial distribution is $P(X_0 = 0) = P(X_0 = 1) = 0.5$.

(a) Find the distribution of X_n.

(b) Find the distribution of X_n when n goes to infinity.



4. There are two types of calls to the Dalseong chicken delivery house. Menu A calls arrive as a Poisson process with rate $\lambda_A = 1$. Menu B calls arrive as an independent Poisson process with rate $\lambda_B = 2$. Let us fix t to be the time that you started to solve this homework.

(a) What is the expected length of the interval that t belongs to? That is, the interval from the last call before t until the first call after t.

(b) What is the probability that t belongs to an AA interval? That is, the first event before, as well as the first event after time t are both of menu A.

(c) What is the probability that between t and t+1, we have exactly two events, one of menu A, followed by one of menu B?

(d) Suppose that a menu B call just arrived. Find the expected time until the end of the first future AA interval? (Hint: Construct a Markov chain with states of A, B and END. The END state is for the end of the first future AA interval. Then, find a pair of equations for the expected time to the END state starting from the A state and the B state.)

5. A weather condition over the satellite link is modeled to be a three-state Markov chain with Sunny, Cloudy, and Rainy states of the transition probability matrix

$$P = \begin{bmatrix} 3/4 & q & 0 \\ 1/2 & r & 1/4 \\ 0 & 3/4 & s \end{bmatrix}$$

(a) Draw a Markov chain showing three states and the transition probabilities.

(b) Find the values of q, r, and s.

(c) Explain that this process has steady-state probabilities π_i. (i.e., the n-step transition probabilities converge to π_i.)