IC666: Discrete Stochastic Processes (Spring 2012)

Assignment 1 (Due: March 29, 2012)

1. Consider a random experiment of tossing a coin three times.

(a) Find the sample space S_1 if we wish to observe the exact sequences of heads and tails obtained.

(b) Find the sample space S_2 if we wish to observe the number of heads in the three tosses.

2. Consider the experiment of tossing a fair coin repeatedly and counting the number of tosses required until the first head appears.

(a) Find the sample space of the experiment.

(b) Find the probability that the first head appears on the *k*th toss.

(c) Verify that P(S) = 1.

3. A lot of 100 semiconductor chips contains 20 that are defective. Two chips are selected at random, without replacement, from the lot.

(a) What is the probability that the first one selected is defective?

(b) What is the probability that the second one selected is defective given that the first one was defective?

(c) What is the probability that both are defective?

4. Suppose that a laboratory test to detect a certain disease has the following statistics. Let

A = event that the tested person has the disease

B = event that the test result is positive

It is known that

$$P(B \mid A) = 0.99 \quad \text{and} \quad P(B \mid \bar{A}) = 0.005$$

and 0.1 percent of the population actually has the disease. What is the probability that a person has the disease given that the test result is positive?

5. Consider the binary communication channel shown in Fig. 1. The channel input symbol X may assume the state 0 or the state 1, and, similarly, the channel output symbol Y may assume either the state 0 or the state 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probabilities p_0, q_0, p_1, and q_1, defined by

$$p_0 = P(y_1 \mid x_0) \qquad \text{and} \qquad p_1 = P(y_0 \mid x_1)$$

$$q_0 = P(y_0 \mid x_0) \qquad \text{and} \qquad q_1 = P(y_1 \mid x_1)$$

where x_0 and x_1 denote the events ($X = 0$) and ($X = 1$), respectively, and y_0 and y_1 denote the events ($Y = 0$) and ($Y = 1$), respectively. Note that $p_0 + q_0 = 1 = p_1 + q_1$. Let $P(x_0) = 0.5$, $p_0 = 0.1$, and $p_1 = 0.2$.

(a) Find $P(y_0)$ and $P(y_1)$.

(b) If a 0 was observed at the output, what is the probability that a 0 was the input state?

(c) If a 1 was observed at the output, what is the probability that a 1 was the input state?

(d) Calculate the probability of error P_e.

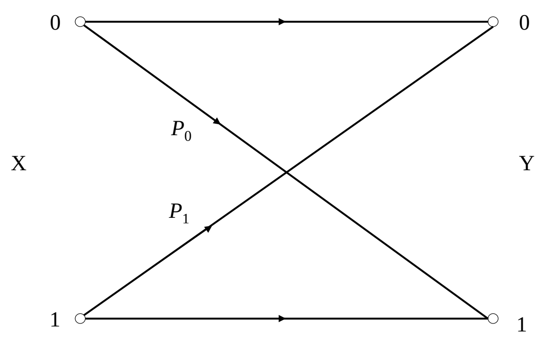

Fig. 1.

6. Let A and B be events in a sample space S. Show that if A and B are independent, then so are (a) A and $\bar{B}$, (b) $\bar{A}$ and B, and (c) $\bar{A}$ and $\bar{B}$.