IC666: Discrete Stochastic Processes (Spring 2012)

Assignment 2 (Due: April 12, 2012)

1. Using a biased coin to make an unbiased decision.

Alice and Bob want to choose between the opera and the movies by tossing a fair coin. Unfortunately, the only available coin is biased (though the bias is not known exactly). How can they use the biased coin to make a decision so that either option (opera or the movies) is equally likely to be chosen?

2. St.Petersburg paradox. You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

3. Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red, independently of the others.

(a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?

(b) Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?

4. Let X and Y be independent random variables that take values in the set {1,2,3}. Let $V = 2X + 2Y$ and $W = X - Y$.

(a) Assume that $P(\{X = k\})$ and $P(\{Y = k\})$ are positive for any $k \in \{1,2,3\}$. Can V and W be independent? Explain. (No calculations needed.)

For the remaining parts of this problem, assume that X and Y are uniformly distributed on {1, 2, 3}

(b) Find and plot $p_V(v)$. Also, determine $E[V]$ and $var(V)$.

(c) Find and show in a diagram $p_{V,W}(v, w)$.

(d) Find $E[V \mid W > 0]$.

(e) Find the conditional variance of W given the event $\{V = 8\}$.

(f) Find and plot the conditional PMF $p_{X|V}(x \mid v)$, for all values.

5. An information source generates symbols at random from. a four-letter alphabet (a, b, c, d} with Probabilities $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{4}$, and $P(c) = P(d) = \frac{1}{8}$. A coding scheme encodes these symbols Into binary codes as follows:

a	0
b	10
c	110
d	111

Let X be the r.v. denoting the length of the code, that is, the number of binary symbols (bits).

(a) What is the range of X?

(b) Assuming that the generations of symbols are independent, find the probabilities P(X = I), P(X = 2), P(X = 3), and P(X > 3).

6. Suppose a discrete r.v. X has the following pmfs:

$$Px(1) = \frac{1}{2} \qquad Px(2) = \frac{1}{4} \qquad Px(3) = \frac{1}{8} \qquad Px(4) = \frac{1}{8}$$

(a) Find and sketch the cdf $F_X(x)$ of the r.v. X.

(b) Find (i) $P(X \leq 1)$, (ii) $P(l < X \leq 3)$, (iii) $P(l \leq X \leq 3)$.