IC666: Discrete Stochastic Processes (Spring 2012)

Assignment 4 (Due: class hour, June 14, 2012)

1. Find the PDF of e^X in terms of the PDF of X. Specialize the answer to the case where X is uniformly distributed between 0 and 1.

2. Two points are chosen randomly and independently from the interval [0,1] according to a uniform distribution. Show that the expected distance between the two points is 1/3.

3. Consider a gambler who at each gamble either wins or loses his bet with probabilities p and 1- p, independent of earlier gambles. When p > 1/2, a popular gambling system, known as the Kelly strategy, is to always bet the fraction 2p - 1 of the current fortune. Compute the expected fortune after n gambles, starting with x units and employing the Kelly strategy.

4. Let X be a random variable that takes the values 1,2, and 3, with the following probabilities:

$$P(X=1)=\frac{1}{2}, \qquad P(X=2)=\frac{1}{4}, \qquad P(X=3)=\frac{1}{4}$$

Find the transform associated with X and use it to obtain the first three moments, $E[X]$, $E[X^2]$, $E[X^3]$.

5. At a certain time, the number of people that enter an elevator is a Poisson random variable with parameter λ. The weight of each person is independent of every other person's weight, and is uniformly distributed between 100 and 200 lbs. Let Xi be the fraction of 100 by which the i th person exceeds 100 lbs, e.g., if the 7th person weights 175 lbs., then X7=0.75. Let Y be the sum of the Xi.

(a) Find the transform associated with Y.

(b) Use the transform to compute the expected value of Y.

(c) Verify your answer to part (b) by using the law of iterated expectations.

6. Let X and Y be two r.v.'s with joint pdf $f_{xy}(x, y)$ and joint cdf $F_{XY}(x, y)$. Let $Z = \max(X, Y)$.

(a) Find the cdf of Z.

(b) Find the pdf of Z if X and Y are independent .

7. Let $Y = aX + b$, where a and b are constants. Show that

(a) $E(Y) = E(aX + b) = aE(X) + b$

(b) $Var(Y) = Var(aX + b) = a^2 Var(X)$