IC666: Discrete Stochastic Processes (Spring 2012)

Assignment 5 (Due: class hour, June 14, 2012)

1. Dave fails quizzes with probability 1/4, independent of other quizzes.

(a) What is the probability that Dave fails exactly two of the next six quizzes?

(b) What is the expected number of quizzes that Dave will pass before he has filed three times?

(c) What is the probability that the second and third time Dave fails a quiz will occur when he takes his eighth and ninth quizzes, respectively?

(d) What is the probability that Dave files two quizzes in a row before he passes tow quizzes in a row?

2. A computer system carries out tasks submitted by two users. Time is divided into slots. A slot can be idle, with probability $p_I = 1/6$, and busy with probability $p_B = 5/6$. During a busy slot, there is probability $p_{1|B} = 2/5$ (respectively, $p_{2|B} = 3/5$) that a task from user 1 (respectively, 2) is executed. We assume that events related to different slots are independent.

(a) Find the probability that a task from user 1 is executed for the first time during the 4^{th} slot.

(b) Given that exactly 5 out of the first 10 slots were idle, find the probability that the 6^{th} idle slot is slot 12.

(c) Find the expected number of slots up to and including the 5^{th} task from user 1.

(d) Find the expected number of busy slots up to and including the 5^{th} task from user 1.

(e) Find the PMF, mean, and variance of the number of tasks from user 2 until the time of the 5^{th} task from user 1.

3. During rush hour, from 8 a.m. to 9 a.m., traffic accidents occur according to a Poission process with a rate of 5 accidents per hour. Between 9 a.m. and 11 a.m., they occur as an independent Poisson process with a rate of 3 accidents per hour. What is the PMF of the total number of accidents between 8 a.m. and 11 a.m.?

4. Customers depart from a bookstore according to a Poisson process with rate λ per hour. Each customer buys a book with probability p, independent of everything else.

(a) Find the distribution of the time until the first sale of a book.

(b) Find the probability that no books are sold during a particular hour.

(c) Find the expected number of customers who buy a book during a particular hour.

5. Transmitters A and B independently send messages to a single receiver in a Poisson manner, with rates of λA and λB, respectively. All messages are so brief that we may assume that they occupy single points in time. The number of words in a message, regardless of the source that is transmitting it, is a random variable with PMF and is independent of everything else.

(a) What is the probability that during an interval of duration t, a total of exactly nine messages will be received?

(b) Let N be the total number of words received during an interval of duration t. Determine the expected value of N.

(c) Determine the PDF of the time from t = 0 until the receiver has received exactly eight three-word messages from transmitter A.

(d) What is the probability that exactly eight of the next twelve messages received will be from transmitter A?

6. Patients arrive at the doctor's office according to a Poisson process with rate λ =1/10 minute. The doctor will not see a patient until at least three patients are in the waiting room.

(a) Find the expected waiting time until the first patient is admitted to see the doctor.
(b) What is the probability that nobody is admitted to see the doctor in the first hour?