W-Simplex: Resilient Network and Control Co-Design under Wireless Channel Uncertainty in Cyber-Physical Systems

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Abstract—In this paper, we propose a resilient network and control co-design architecture, Wireless-Simplex (W-Simplex), which can guarantee control performance by adaptively tuning the network and control parameters against wireless channel uncertainty in cyber-physical systems (CPS). To the best of our knowledge, there has been no study on resilient network and control co-design in response to the unreliable wireless channel. Our key observation is that rate adaptation may cause significant degradation in control performance or even system instability. This performance degradation is contrary to the intuition that rate adaptation provides a reliable link under wireless channel uncertainty. We explain the cause of this phenomenon and resolve the situation by proposing a resilient co-design algorithm in an optimization framework. Our simulation study with ns-2 shows the effectiveness of the proposed scheme for providing resilience of CPS against wireless channel uncertainty.

I. INTRODUCTION

Recently, the convergence of cyber and physical spaces [1]–[4] has turned traditional embedded systems into cyber-physical systems (CPS), which are primarily characterized by tight integration between computational and physical systems by means of networking. In CPS, various embedded devices with computational components are networked to monitor, sense, and actuate physical elements in the real world. Consequently, CPS can be effectively decomposed into three major parts; cyber elements representing software, physical entities consisting of various real-world systems, and networks connecting the cyber and the physical world.[1]

Among many desirable characteristics of CPS, resilience is one of the most crucial features [5]. Broadly speaking, resilience is to maintain an accepted level of operational normalcy in response to disturbances [6]. By considering the three elements of CPS, the major challenges for resilient CPS are as follows: (i) software errors in the cyber systems; (ii) model uncertainty and disturbances in the physical systems; and (iii) wireless channel uncertainty in networks.[2]

In the literatures, (i) is studied in [7], in which the Simplex architecture is proposed for resilience against software errors. More recently, the L1Simplex architecture is presented in [8] to address both software and physical failures, which resolves (i) and (ii) at the same time. However, existing studies on network and control co-design do not explicitly address the issue of wireless channel uncertainty of (iii). Consequently, we need a resilient network and control co-design architecture against unreliable wireless channel.

In this paper, we pay attention to wireless channel uncertainty and propose a resilient network and control co-design architecture, which can make the system resilient against the adverse effect of wireless channel uncertainty. In particular, our main contributions are as follows:

- We show that rate adaptation for maintaining a robust network link under wireless channel uncertainty may cause degradation of control performance or even instability.
- In order to overcome performance degradation, we incorporate the PHY data rate into the network and control co-design formulation.
- We propose Wireless-Simplex (W-Simplex) that can adaptively tune the network and control parameters for resilience against wireless channel uncertainty.

In the meantime, in wireless communications, it is well known that adaptive modulation and coding or so-called rate adaptation can make the communication link robust against a time-varying, uncertain wireless channel. However, in terms of control performance in networked control, rate adaptation may result in severe degradation or even system instability, which has never been properly addressed in any previous work. Consequently, we need to understand this phenomenon and resolve the situation for providing resilience in CPS.

II. MOTIVATION: WHAT HAPPENS TO CONTROL PERFORMANCE UNDER WIRELESS CHANNEL UNCERTAINTY?

In this section, we show control performance degradation of a wireless networked control system due to wireless channel uncertainty. First, Fig. 1(a) shows control performance without any rate adaptation when the wireless channel becomes poor.[3] In the figure, the y-axis corresponds to the controlled system output, which is rather a vehicle for information exchange between the two.

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2We do not explicitly consider resilience against attacks, which will be a subject of future work.

3Details of the simulation environment can be found in Section V.
in the steady state. In this case, since the packet reception ratio is seriously reduced because of the lower signal to interference noise ratio (SINR) as soon as the channel becomes worse, it is obvious that control performance shows instable behavior as shown in Fig. 1(a).

Now, we consider the situation with rate adaptation when the data rate changes from 11 Mb/s to 1 Mb/s in order to maintain a reliable link against wireless channel degradation. In this case, even though the link is kept reliable without any significant packet drops, the simulation result in Fig. 1(b) still shows severe oscillating instability. This phenomenon is contrary to our intuition that rate adaptation provides a robust link against wireless channel uncertainty. In the following sections, we will give a more rigorous explanation on this phenomenon, and present a resilient co-design framework.

III. PROBLEM FORMULATION

In the traditional digital control, a smaller sampling period gives better control performance though the performance saturates at a certain point. In the meantime, the performance of networked control initially improves as the sampling period decreases. However, since network traffic is inversely proportional to the sampling period, more contention will occur as the sampling period becomes smaller. Consequently, at a certain point, the network delay will become significant enough to degrade the control performance and may even cause instability. This performance comparison is illustrated in Fig. 2. Consequently, unlike traditional digital control, we need to carefully co-design network and control in networked control systems in order to satisfy both network and control performance.

We give the conventional optimization formulation of network and control co-design in cyber-physical systems (CPS). The overall structure of CPS is described in Fig. 3. There are \( N \) physical systems and \( N \) controllers, where \( N \) is an integer. The physical systems periodically send sensed data to the controllers over a wireless network.\(^4\)

Each controller \( C_n, \ n = 1, \ldots, N \) has a sampling period of \( s_n \), which defines how often sensed data is sampled and transmitted to the controller. In addition, network delay \( d_n, \ n = 1, \ldots, N \) of each feedback loop is represented as \( d_n(s, V, \delta) \), where \( s \) and \( V \) are respectively the set of the sampling periods and the network parameters such as contention window size and retransmission count, and \( \delta \) includes the network setup such as network topology and the number of nodes.

In fact, the conventional optimization formulation does not include rate adaptation, which is the cause of the control performance degradation shown in Fig. 1(b). Without any rate adaptation, the packet loss probability will become larger with a poor channel condition, which may result in degradation of control performance or system instability as shown in Fig. 1(a). In case of rate adaptation, let \( s^* \) and \( V^* \) denote the solution to optimization formulation. When rate adaptation lowers the PHY data rate due to a poor channel condition, the region of the sampling period for system stability will become smaller because of increased network contention, as shown in Fig. 4.\(^5\) This phenomenon occurs because the network delay \( d_n \) is a decreasing function of the data rate. Consequently, for system resilience, we need to explicitly include the effect of wireless channel uncertainty.

Hence, the network and control co-design under wireless channel uncertainty can be formulated in an optimization framework. Let \( r_n \) denote the PHY data rate of the feedback loop \( n \). Then, among a finite set of values, \( r_n \) is selected depending on random channel condition denoted by \( \xi \), i.e.,

\(^4\)Here, we assume that controllers are directly connected to the actuators of the physical systems while sensors are connected to the controllers over a network as in other related work [9]–[11].

\(^5\)It should be noted that Fig. 4 is a simulation result while Fig. 2 is a conceptual illustration. Details of the simulation environment can be found in Section V.
\[ r_n = r_n(\xi) \]

Consequently, for a given value of \( r_n \), network and control co-design can be formulated as the following constrained optimization:

\[
\begin{align*}
\min_{s, \mathbf{V}} & \quad F(s, \mathbf{V}, r_n(\xi), \delta) \\
\text{s.t.} & \quad J_n(s_n, d_n(s, \mathbf{V}, r_n(\xi), \delta)) \leq J_n^\text{th}, \quad n = 1, \ldots, N.
\end{align*}
\]

(1)

Here, \( F(s, \mathbf{V}, \delta) \) denotes an objective function for network performance such as network energy consumption. \( J_n \) represents control cost of the \( n \)-th physical system, and \( J_n^\text{th} \) is a pre-specified threshold of control cost \( J_n \).

IV. W-SIMPLEX: RESILIENT ARCHITECTURE FOR NETWORK AND CONTROL CO-DESIGN

In this section, we propose a resilient architecture that can adaptively tune the network and control parameters against wireless channel uncertainty. In a nutshell, we adopt the idea of the Simplex architecture [7], which is a switching architecture between a high performance algorithm and high assurance one for resilience under software errors.

A. Wireless-Simplex (W-Simplex): Network and Control Co-Design against Wireless Channel Uncertainty

Similarly as in the Simplex architecture, we consider a switching architecture for the PHY data rate \( r_n(\xi) \). The basic idea is as follows: The values of \( s \) and \( \mathbf{V} \) need to be appropriately tuned according to \( r_n(\xi) \). Hence, once the data rate is determined according to the channel condition, \( s \) and \( \mathbf{V} \) are switched to proper values in order to satisfy both the network and control performance given in (1). In other words, if \( r_n(\xi) \) is selected in a finite set of \( \{r^1, \ldots, r^D\} \), we solve (1) to get the solution of \( s^* \) and \( \mathbf{V}^* \) for each \( r^i \), \( i = 1, \ldots, D \) in advance. Then, for each value of \( r_n(\xi) \), \( s \) and \( \mathbf{V} \) are switched to the corresponding \( s^* \) and \( \mathbf{V}^* \). The overall W-Simplex architecture is given in Fig. 5.

Now, the remaining issue is how to obtain models for the control cost, network delay, and network performance. In the subsequent sections, we derive these models.

B. Control Cost

In this section, in order to solve a given constrained optimization in (1), we model the control cost \( J_n \) of the \( n \)-th networked control system. Since the analysis applies for all the systems \( n = 1, \ldots, N \), we omit the subscript \( n \) for simplicity.

It is assumed that the physical system is modeled as a single-input single-output linear system. Specifically,

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B_1 u(t) + B_2 w(t), \\
y(t) &= C x(t), \\
u(t) &= K x(t),
\end{align*}
\]

(2)

where \( x(t) \in \mathbb{R}^m \) is the state of the physical system, the dimension of which is \( m \), \( u(t) \in \mathbb{R} \) is the control input, \( w(t) \in \mathbb{R} \) is disturbance, and \( y(t) \in \mathbb{R} \) is the output, respectively. The disturbance \( w(t) \) is assumed to be a zero-mean white Gaussian process whose power spectral density is given by a constant \( W \). The matrices \( A, B_1, B_2, \) and \( C \) define the physical system dynamics, and are in appropriate dimensions. A state feedback control is assumed with gain matrix \( K \), designed to yield \( (A-B_1K) \) to be Hurwitz matrix for stability.

Since the system is driven by a random disturbance, we define the control performance by the expected value of \( y(t)^2 \), i.e.,

\[
J = E[y^2(t)] = CE[x(t)x(t)^T]C^T,
\]

that gives a measure of the deviation of \( y \) from a desired value of 0. This can be achieved by calculating the covariance of the state, i.e., \( E[x(t)x(t)^T] \). Measuring the deviation of \( y \) from any nonzero desired value can also be done by an appropriate coordinate change. Qualitatively speaking, the control cost \( J \) is the variance of output in control. A larger value of \( J \) means that the system state deviates more around the reference in the steady state. Hence, the larger the value of \( J \) is, the worse the control performance becomes.

Discretizing the system (2) with sample period \( s \) yields the following difference state equation:

\[
\begin{align*}
x[k + 1] &= \Phi x[k] + \Gamma u[k] + \gamma[k], \\
y[k] &= C x[k], \\
u[k] &= K x[k],
\end{align*}
\]

where \( \Phi = e^{As}, \quad \Gamma = \int_0^s e^{A\tau} d\tau B_1, \) and \( \gamma[k] = \int_0^s e^{A(s-\tau)} B_2 w(\tau) d\tau. \)
are written as
\[
x[k + 1] = \Phi x[k] + \Gamma K x[k - k_d] + \gamma[k], \\
y[k] = C x[k],
\] (3)

where \( \Phi_{CL} = \Phi - \Gamma K \) is a closed loop state matrix.

The network induced delay is denoted by \( d_n \) in Section III. As it may be natural for a discrete-time control system, we assume that \( d_n \) is a multiple of the sampling period \( s \) and given by \( d_n = k_d \cdot s \) for a positive integer \( k_d \). Then, the control system including the delay can be written as
\[
x[k + 1] = \Phi x[k] - \Gamma K x[k - k_d] + \gamma[k], \\
y[k] = C x[k],
\]

which in turn can also be rewritten as follows:
\[
X[k + 1] = \Phi_d X[k] + F \gamma[k], \\
y[k] = C_0 X[k],
\] (4)

where \( X[k] = [x^T[k] \ x^T[k - 1] \ \cdots \ \cdots \ x^T[k - k_d]]^T, \ F = [I_{m \times m} \ 0_{m \times m} \ \cdots \ \cdots \ 0_{m \times m}], \ C_d = [C_0 0_{1 \times m} \ \cdots \ 0_{1 \times m}], \) and
\[
\Phi_d = \left[ \Phi - \Gamma K \right]_{0 \times m \times m} \\
I_{m \times m} \\
\vdots \\
0_{m \times m} \ I_{m \times m} \ 0_{m \times m}
\]

Denote the state covariance matrix of the system (3) without delay \( (d = 0) \) by
\[
P[k] = E[x[k]x^T[k]].
\]

Then, it is obtained by solving the following Lyapunov equation
\[
P[k + 1] = \Phi_d P[k] \Phi_d^T + Q,
\]
where \( Q = \int_0^s e^{\Phi \tau} B_2 W B_2^T e^{\Phi^T \tau} \, d\tau \).

If all eigenvalues of the matrix \( \Phi_{dCL} \) are inside the unit circle and there exists the positive definite matrix \( P \) which is the solution of
\[
P = \Phi_{dCL} P \Phi_{dCL}^T + Q,
\]
then \( P[k] \) is given by \( P \) in the steady state. Thus, the control cost \( J \) is defined as follows:
\[
J = CPC^T.
\]

For the case of the system of (4), the equation for the covariance matrix \( P_d \) (the subscript \( d \) denotes the delay) is given by
\[
P_d[k + 1] = \Phi_d P_d[k] \Phi_d^T + Q_d, \tag{5}
\]
where \( P_d[k] = E[X[k]X^T[k]] \), and \( (k_d + 1)m \) dimensional square matrix \( Q_d \) is given by
\[
Q_d = \begin{bmatrix}
Q & 0_{m \times m} & \cdots & 0_{m \times m} \\
0_{m \times m} & 0_{m \times m} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0_{m \times m} & \cdots & \cdots & 0_{m \times m}
\end{bmatrix}.
\]

In (5), if all eigenvalues of the matrix \( \Phi_d \) are inside the unit circle, and there exists the positive definite matrix \( P_d \) which is the solution of
\[
P_d = \Phi_d P_d \Phi_d^T + Q_d,
\]
then the control cost is given by
\[
J_d = C_d P_d C_d^T.
\]

C. Network Delay Model

In this section, we model the network delay in a similar manner as in related co-design studies [9], [10], [16]. Unlike homogeneous network models used in the previous work, we adopt a heterogeneous network model, in which each network node can have different values of network parameters [17].

We consider IEEE 802.11 WLAN as the network between the physical systems and the controllers. Let the network consist of \( N \) nodes with packet arrival rates of \( \lambda_n, n = 1, \ldots, N \). The utilization and collision probability of each node are respectively denoted by \( \rho_n \) and \( p_n, n = 1, \ldots, N \). Then, the average backoff window of node \( n \) can be represented as
\[
W_n = \frac{1 - \rho_n - p_n (2p_n)^m}{1 - 2p_n} \frac{CW_{\text{min}}}{2},
\]
where \( m \) is the maximum number of retransmission and \( CW_{\text{min}} \) is the minimum of the contention window size.

The collision probability of node \( n \) in a slot is given by
\[
p_n = 1 - \prod_{i=1}^{N} \left( 1 - \rho_i \frac{1 - 2p_i}{1 - p_i} \frac{2}{2(1 - p_i)} \right). \tag{6}
\]

Here, the utilization of node \( n, \rho_n \) is expressed as follows:
\[
\rho_n = \sum_{i=1}^{N} \lambda_n \rho_i \left[ T_{S_i} + T_{C_i} \frac{p_i}{2(1 - p_i)} \right] + \lambda_n \left[ \frac{W_n + T_{S_n} + T_{C_n} \frac{p_n}{2(1 - p_n)}}{2} \right], \tag{7}
\]

where \( T_{S_i} \) and \( T_{C_i} \) are the length of node \( i \)'s packet and collision in the unit of backoff slots, respectively. We can now numerically solve a system of equations (6) and (7) because they are \( 2N \) equations with \( 2N \) variables.

Now, the probability generating function of the final service time, \( B_n(z) \) and the probability generating function of the number of packets which arrive in a slot, \( A_n(z) \) can be obtained by the \( CW_{\text{min}}, m, \rho_n \), and \( p_n \) [17].

Here, the mean of the network delay \( d_n \) is given by
\[
d_n = B_n'(1) + \frac{A_n'(1) B_n'(1) + A_n'(1) B_n'(1)}{2 A_n'(1)[1 - A_n'(1) B_n'(1)]}.
\]

D. Network Performance Objective

Now, we consider the network-wide energy consumption as the objective function \( F(s, V, r_i(\xi, \delta)) \). Let \( SE \) and \( QNE \) denote the event that a slot is empty and the queue is
which allows for backward compatibility with older 802.11 standards. The modulation scheme in the channel is ERP-DSSS/CCK, not empty, respectively. Then, \( P[SE] \) can be represented as

\[
P[SE] = 1 \times (1 - \rho) + \rho P[SE][QNE].
\]

Here, \( P[SE][QNE] \) can be further approximated by \((W - 1)/W\). Therefore, we have

\[
P[SE] = (1 - \rho) + \rho \frac{W - 1}{W} = 1 - \frac{\rho}{W}.
\]

Consequently, network energy consumption of all the nodes can be calculated as

\[
F = \sum_{i=1}^{N} \frac{\rho_i}{W_i}.
\]

V. Performance Evaluation

In this section, we carry out simulation study to validate the effectiveness of the proposed architecture of W-Simplex.

A. Simulation Setup

Our simulator is implemented in \textit{ns-2}. By putting everything in Section IV together, we implement an iterative algorithm for solving (1) for each value of the data rate \( r_i \in \{r^1, \cdots, r^D\} \). In order to focus on the control performance rather than the dynamics of a specific rate adaptation algorithm, we implement ideal rate adaptation that immediately adjusts the data rate with perfect knowledge on the channel condition change. In this manner, we do not need to consider the transient behavior of rate adaptation, and can focus on the control performance.

In addition, to clearly show the effect of rate adaptation, we use IEEE 802.11g instead of more recent standards. IEEE 802.11g includes two sets of mandatory modulation schemes which are ERP-DSSS/CCK and ERP-OFDM. We consider the modulation scheme in the channel is ERP-DSSS/CCK, which allows for backward compatibility with older 802.11 radios. The value of \( L \), which represents the time to transmit a packet, is defined as

\[
L = PLCP_P + PLCP_H + \frac{\text{Packet size}}{\text{PHY rate}}.
\]

where \( PLCP_P \) and \( PLCP_H \) represent the length of time required to transmit the PLCP preamble and the PLCP header. In case of ERP-DSSS/CCK, the PHY data rate includes 1, 2, 5.5, 11 Mb/s.

B. Control Performance under Uncertain Wireless Channel

In Section II, the control system suffers from performance degradation and oscillating instability when the wireless channel condition becomes poor, even though ideal rate adaptation is used. Here, we show the performance of the proposed architecture of W-Simplex.

We set the the control parameter \( s \) as the set of the sampling periods of all the control system, and the network parameter \( V \) as the set of the contention window values of all the network nodes.

As a control performance metric, we use \( e^2 \), the square of the control error \( e \), which is the difference between the system output and the reference. Simply speaking, smaller \( e^2 \) implies better control performance. We take into account two types of physical systems, \( S_1 \) and \( S_2 \), which have different system parameters. The system parameters of \( S_1 \) are such that \( A = [0 1; 0 0], B_1 = B_2 = [0; 1], C = [1 0], K = [-15 - 8], W = 1 \) and the system parameters of \( S_2 \) are such that \( A = [0 1; 0 0], B_1 = B_2 = [0; 1], C = [1 0], K = [-24 - 10], \) and \( W = 1 \). We assume 20 physical systems, among which the half are \( S_1 \) and the rest half are \( S_2 \). It should be noted that dynamics of both \( S_1 \) and \( S_2 \) are simple, yet effective enough to show unstable behavior under delayed feedback. In fact, the physical system used in simulation is a double integrator, which is representative of many electro-mechanical systems. It will be a subject of future work to investigate the performance of W-Simplex with more specific physical systems.

Figure 6(a) shows the control performance with a fixed sampling time of 0.05 s when the data rate changes from 11 Mb/s to 2 Mb/s due to a poor channel condition at 900 seconds. The upper and lower figures respectively show the control performance of each type of physical systems. As already expected from Fig. 4, the control performance for both types of systems becomes unstable under the data rate of 2 Mb/s.

Now, we show the performance of W-Simplex in Fig. 6(b). The sampling periods of \( S_1 \) and \( S_2 \) are respectively tuned to 73.4 ms and 84.9 ms under the PHY data rate of 2 Mb/s. The control cost \( e^2 \) of both systems remain quite small values, which implies resilient control performance under wireless channel uncertainty. Both types of the systems shows good control performance.

Finally, we show the control cost as a function of the sampling period and the data rate in Fig. 7, where we can clearly discover that the sampling periods for stable control performance become larger for lower data rates. Overall, the sampling period is a determining factor for stability of the physical systems while the effect of the network parameter of the contention window is virtually negligible. In fact, the solutions of (1) give that the optimal network parameter of the contention windows remain the same as a value of 32, which implies that the sampling period dominates the control performance as shown in Fig. 7.

![Fig. 6. Control cost $e^2$ when data rate changes from 11 Mb/s to 2 Mb/s (upper: $S_1$, lower: $S_2$).](image-url)
VI. RELATED WORK

There exist extensive work on networked control systems. Hence, we introduce some of recent studies. Interested readers may refer to [13]–[15] for further information. Control design for networked control systems has been studied in [18], [19]. However, in these studies, the networks are simply modeled as a source of delay and packet loss, and the details of network protocols are not properly considered.

An optimization formulation for network and control co-design is proposed in [9], which corresponds to the conventional co-design framework without wireless channel uncertainty. In [16], a distributed adaptive algorithm is proposed for minimizing power consumption while guaranteeing a given packet reception probability and delay constraints in CPS. The joint optimization of controllers and communication systems is introduced in [20], which encompasses efficient abstractions of both systems. These recent studies give network and control co-design methods for CPS. However, the effect of wireless channel uncertainty on quality of control is not considered.

The Simplex architecture is proposed in [7], which switches between a high performance algorithm and a high assurance one for resolving software errors. The L1Simplex architecture is introduced in [8], which can address both software and physical failures. In [21], the NetSimplex is presented, which extends the original Simplex architecture in [8] to a networked control environment.

In summary, even though there have been substantial research on network and control co-design, resilient co-design against wireless channel uncertainty has not been properly addressed so far.

VII. CONCLUSION

In this paper, we have proposed a high-level switching architecture of W-Simplex, which can adaptively tune the network and control parameters for providing resilience against wireless channel uncertainty.

There remain several important issues for future work. It will be an interesting subject of future work to extend the proposed framework by incorporating various cyber-physical attack scenarios [22]. Another direction of future research is to further investigate the mathematical structure of the co-design optimization problem.

REFERENCES