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Stochastic analysis of packet-pair probing for network bandwidth estimation

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Abstract

In this paper, we perform a stochastic analysis of the packet-pair technique, which is a widely used method for estimating the network bandwidth in an end-to-end manner. There has been no explicit delay model of the packet-pair technique primarily because the stochastic behavior of a packet pair has not been fully understood. Our analysis is based on a *novel* insight that the transient analysis of the G/D/1 system can accurately describe the behavior of a packet pair, providing an *explicit* stochastic model. We first investigate a single-hop case and derive an analytical relationship between the input and the output probing gaps of a packet pair. Using this single-hop model, we provide a multi-hop model under an assumption of a single tight link. Our model shows the following two important features of the packet-pair technique: (i) The difference between the proposed model and the previous fluid model becomes significant when the input probing gap is around the characteristic value. (ii) The available bandwidth of any link after the tight link is *not observable*. We verify our model via `ns`-2 simulations and empirical results. We give a discussion on recent packet-pair models in relation to the proposed model and show that most of them can be regarded as special cases of the proposed model.

Keywords: Packet-pair technique; Bandwidth estimation; M/D/1 queue; Transient analysis

1. Introduction

To improve the performance of the Internet applications in various aspects, it is crucial to understand and exploit useful properties of an end-to-end path. For example, the *available bandwidth* is one such characteristic, which can be used in numerous ways [1,2]: in many bandwidth-sensitive applications such as peer-to-peer applications and on-demand multimedia streaming applications, a client will benefit if it is connected to a peer via a path with sufficient available bandwidth. Further, the performance of overlay networks can be improved if we can estimate the available bandwidth between nodes.

Now, we formally introduce two important characteristics of an end-to-end path: the *bottleneck capacity* and the *available bandwidth*. Consider a path $\mathscr{P}$ as a set of links that forward packets from a sender $\mathscr{S}$ to a receiver $\mathscr{R}$. Assume that the path $\mathscr{P}$ is fixed and unique for the duration of the

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measurement. Let M denote the number of links in $\mathcal{P}$, then each link i, $i = 1, \ldots, M$ transmits packets with a constant rate of C_i Mb/s, which is referred to as the *link capacity*. Then, the *bottleneck capacity* of path $\mathcal{P}$ is

$$C_{\mathcal{P}} = \min_{i=1,\ldots,M} C_i$$

and the link with $C_{\mathcal{P}}$ is called the *narrow link*. Also, if we let u_i denote the utilization of link i over a certain interval, the available bandwidth of path $\mathcal{P}$ is

$$A_{\mathcal{P}} = \min_{i=1,\ldots,M} [C_i(1 - u_i)]$$

and the link with $A_{\mathcal{P}}$ is called the *tight link*. Note that the tight link may be different from the narrow link in general. A more detailed explanation of the bottleneck capacity and the available bandwidth can be found in [2].

There has been a lot of research on the problem of measuring the bottleneck capacity and the available bandwidth in an end-to-end manner. However, designing an accurate mechanism to measure the network bandwidth in an end-to-end path is challenging since the Internet is a very complex dynamic system. A general way of active bandwidth measurement is to inject *probing* packets into an end-to-end path and observe their behavior to estimate the bottleneck capacity and the available bandwidth. One of the most popular mechanisms is the *packet-pair* technique [3–5], in which a source sends multiple packet pairs to a receiver. Each packet pair usually consists of two probing packets of the same size. The inter-packet dispersion between these two probing packets changes according to path characteristics such as *link capacities* and *cross-traffic*. (Note that cross-traffic represents all the traffic in the path except the probing packets.) Hence, the relationship between the inter-packet dispersion at the sender and that at the receiver is exploited to estimate the end-to-end network bandwidth. Hereafter, we use the input/output probing gap and the inter-packet dispersion at the sender/receiver interchangeably.

The packet-pair technique has been extensively studied for network bandwidth estimation. Actually, many tools for measuring the bottleneck capacity and the available bandwidth are based on the packet-pair technique [1,2,6–13]. However, most of these measurement methods rely on qualitative aspects or empirical results instead of an analytical model. Hence, the establishment of an accurate model of the packet-pair technique is of fundamental importance for designing efficient bandwidth estimation tools. Recently, a deterministic packet-pair model has been developed in [8]. Yet, this deterministic model assumes fluid cross-traffic with a constant rate and consequently ignores the stochastic nature of cross-traffic. More recently, Liu et al. [14] has given a stochastic analysis of packet pair/train probing, which provides upper and lower bounds of the relationship between the input and output probing gaps using a sample-path analysis under 'mild' assumptions.

The main contribution of this paper is a derivation of a *stochastic* model of the packet-pair technique, which accurately captures the stochastic nature of cross-traffic. Unlike [14], we derive an *explicit* model that shows a mathematical relationship between the input and the output probing gaps. There has been no explicit delay model of the packet-pair technique, mainly because the stochastic behavior of a packet pair has not been fully understood. Our analysis is based on a *novel* insight that the relationship between the input and the output probing gaps of a packet pair is governed by the transient behavior of a queueing system. Hence, under a reasonable assumption of Poisson cross-traffic, we give a stochastic delay model of the packet-pair technique via the transient analysis of the M/D/1 system.

First, we investigate a single-hop case and derive a stochastic delay model, which describes the analytical relationship between the input and the output probing gaps of a packet pair. Then, we examine a multi-hop case and derive an explicit multi-hop model under an assumption of a single tight link. We validate the proposed model via ns-2 simulations and empirical results. Our model shows the following two important features of the packet-pair technique: (i) The difference between the proposed model and the previous fluid model becomes significant when the input probing gap is around the characteristic value. (ii) The available bandwidth of any link after the tight link is *not observable*. Based on our analysis, we discuss recent packet-pair models in relation to the proposed model and show that most of them can be regarded as special cases of the proposed model.

The rest of the paper is organized as follows: In Section 2, we provide preliminaries for the problem addressed in the paper. In Sections 3 and 4, we introduce a stochastic model, which can capture the essential stochastic nature of cross-traffic. In Section 5, we validate the stochastic model through ns-2 simulations and empirical results. Based on the analysis, we present important features of the packet-pair technique and discuss several recent packet-pair models in relations to the proposed model in Section 6. Finally, the conclusion follows in Section 7.

2. Preliminaries

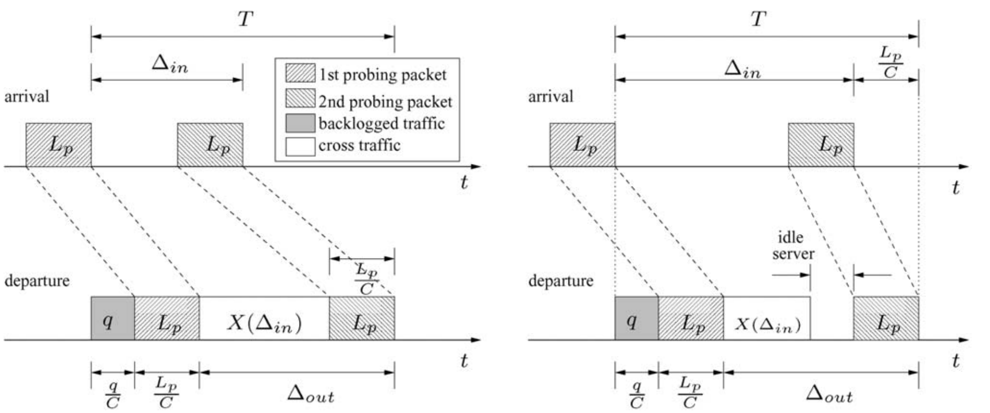
In this section, we explain how the packet-pair technique works in detail. Then, we introduce the deterministic packet-pair model in [8]. A single-hop deterministic model of the packet-pair technique was proposed in [8] under the assumption of fluid cross-traffic with a constant rate and consequently, the stochastic nature of cross-traffic was ignored.

The packet-pair technique works as follows: A source sends *packet pairs* to a receiver. Each packet pair consists of two probing packets, usually of the same size. The time dispersion between these two packets changes according to path characteristics such as *link capacities* and *cross-traffic* as they traverse the path. Hence, the relationship between the inter-packet dispersion at the sender and that at the receiver is exploited to obtain information on network bandwidth. The inter-packet dispersion Δ_{in} at the sender is defined as the interval between the departure of the first probing packet from the sender and that of the second probing packet. In a similar manner, the inter-packet dispersion Δ_{out} at the receiver is defined as the interval between the arrival of the first probing packet at the receiver and that of the second probing packet. Our main concern here is how to establish an analytical relationship between Δ_{out} and Δ_{in}.

Consider a single link with capacity C. The sender at one end of the link transmits a packet pair to the receiver at the other end. The size of each probing packet is L_{p} bytes. Let t_0 denote the time when the first probing packet arrives at the queue. Also, let q denote the amount of cross-traffic already in the queue when $t = t_0$. Fig. 1 illustrates the arrival and departure of a packet pair on a single link. In the figure, T denotes the interval between the arrival of the first probing packet and the departure of the second probing packet, and $X(t_0, \Delta_{\text{in}})$ denotes the amount of cross-traffic arrived during $[t_0, t_0 + \Delta_{\text{in}}]$.

Under the assumption of fluid cross-traffic, the second probing packet will see the server busy or idle according to whether the server has been fully utilized or not during $[t_0, t_0 + \Delta_{\text{in}}]$. The case of a busy server is given in Fig. 1(a). Similarly, the case of an idle server is presented in Fig. 1(b). Note that $X(\Delta_{\text{in}})$ is used instead of $X(t_0, \Delta_{\text{in}})$ in the figure since $X(t_0, \Delta_{\text{in}})$ is independent of t_0 under the fluid assumption. Also, we should note that, without the assumption of fluid cross-traffic with a constant rate, it will be the last busy period in $[t_0, t_0 + \Delta_{\text{in}}]$ that determines whether the server is busy or idle when the second probing packet arrives. The server will be busy if the second probing packet arrives before the end of the last busy period. On the other hand, if the last busy period ends before the second probing packet arrives, the server will be idle.

Under the assumption of fluid cross-traffic with a constant rate r, $X(t_0, \Delta_{\text{in}})$ yields a deterministic value of $r\Delta_{\text{in}}$ and the following relationship between Δ_{out} and Δ_{in} can be derived depending on whether the link is fully utilized or not [8]:



(a) When the second probing packet sees the server busy (b) When the second probing packet sees the server idle

Fig. 1. Behavior of a packet pair in a single link with fluid cross-traffic.

$$\Delta_{\text{out}} = \begin{cases} \frac{r}{C}\Delta_{\text{in}} + \frac{L_p}{C}, & \Delta_{\text{in}} \leqslant \Delta^* \left(= \frac{L_p+q}{C-r}\right), \\ \Delta_{\text{in}} - \frac{q}{C}, & \text{otherwise}. \end{cases} \quad (1)$$

From (1), we know that the graph of $(\Delta_{\text{in}}, \Delta_{\text{out}})$ consists of two line segments and the change of slope occurs at $\Delta_{\text{in}} = \Delta^* \left(= \frac{L_p+q}{C-r}\right)$, which is termed as the *characteristic value*. Further, if we let Δ^c denote the value of Δ_{in} such that $\Delta_{\text{in}} = \Delta_{\text{out}}$, then $\Delta^c = \frac{L_p}{C-r}$, which is termed as the *critical value*. Note that we can obtain the available bandwidth A once we know Δ^c by $A = C - r = L_p/\Delta^c$. Since $q \approx 0$ under the assumption of fluid cross-traffic, $\Delta^c \approx \Delta^*$ and we further have $A = C - r \approx L_p/\Delta^*$. Actually, many measurement algorithms seek the available bandwidth A by estimating the characteristic value Δ^* [1,8,10,13]. However, we can easily suspect that considerable modeling error is introduced in (1) by the assumption of fluid cross-traffic with a constant rate. In this paper, we consider $X(t_0, \Delta_{\text{in}})$ as a stochastic process and derive a model that depicts the relationship between Δ_{out} and Δ_{in} more accurately. Then, we discuss recent packet-pair models in relation to the proposed model.

3. Packet-pair delay model: single-hop case

In this section, our primary concern is to derive a single-hop relationship between the input and the output probing gaps under an assumption of stationary Poisson cross-traffic. Based on this assumption, we show that the transient analysis of the M/D/1 system can accurately describe the behavior of a packet pair. By adopting the M/D/1 analysis, we only consider packets of the same size in cross-traffic. Since the packet size distribution of the Internet traffic is known to be multi-modal [15], $M^{[X]}/D/1$ analysis which allows batch arrivals would be more complete [16]. However, the M/D/1 analysis plays an essential role in the $M^{[X]}/D/1$ analysis. Also, the M/D/1 analysis by itself gives a valuable insight, namely that the queueing theory can be successfully applied to the modeling of the packet-pair technique for bandwidth measurement. For these reasons, the focus here is on the M/D/1 analysis of the packet-pair technique.

Here, we adopt the transient analysis of the M/D/c queueing system in [17]. First, we give our rationale as to how the behavior of a packet pair can be described by the transient queueing analysis. Then, based on the M/D/1 analysis, we develop the state vector in a transient regime and the concept of the virtual waiting time in order to depict the waiting time of the second probing packet. Finally, we give a stochastic delay model of the packet-pair technique.

3.1. The key idea of M/D/1 modeling

Here, we assume that the cross-traffic is a stationary Poisson process in the time scale of interest. The stationary assumption for the Internet traffic is typical in the literature and is known to be empirically acceptable over intervals of several minutes up to a few hours [18,19].

When the first probing packet arrives at the queue at $t = t_0$, assume that there are q_1 packets already in the queue and one in service with the residual service time d. (If there is no packet in service at $t = t_0$, $d = 0$.) In a similar manner, assume that there are q_2 packets already in the queue when the second probing packet arrives at the queue. Let W_1 and W_2 denote the waiting times (in queue) of the first and the second probing packets, respectively. Since both the service times of the first and the second probing packets equal to L_p/C, we have

$$\begin{aligned} \Delta_{\text{out}} - \Delta_{\text{in}} &= (W_2 + L_p/C) - (W_1 + L_p/C) \\ &= W_2 - W_1. \end{aligned} \quad (2)$$

The key insight is that, from the viewpoint of the second probing packet, W_2 is the transient waiting time in queue of the M/D/1 system with initially $q_1 + L_p/L_c + 1$ packets, where L_c is the packet size of cross-traffic.[1] The rationale is as follows: Just after the arrival of the first probing packet, i.e., $t = t_0^+$, there are a total of $q_1 + L_p/L_c + 1 =: N_0$ packets in the system including one possibly in service. Now, the overall system can be considered as the M/D/1 queue with initially N_0 packets in the system, which begins to evolve at $t = t_0$. Hence, without loss of generality, let $t_0 = 0$. Then, the waiting time W_2 of the second probing packet exactly corresponds to that of a packet which arrives at the M/D/1 system at $t = \Delta_{\text{in}}$. Consequently, the problem now becomes how to find the transient waiting time of the M/D/1 system at $t = \Delta_{\text{in}}$ initially holding N_0 packets.

[1] In order to make the problem more tractable, it is assumed that L_p/L_c is an integer and the round-off effect is ignored.

3.2. Waiting time distribution of the second probing packet

First, we introduce the state vector of the M/D/1 system. Let $\pi_j(t)$ denote the probability of the system holding j packets at time t. Then, the state vector $\pi(t) =: (\pi_0(t), \pi_1(t), \pi_2(t), \ldots)$ can be obtained as follows:

Lemma 1. (i) *When* $t \in (0, D]$ *where* $D = L_c/C$,

$$\pi_j(t) = \begin{cases} \frac{(\lambda t)^{j-N_0+K(t)}}{(j-N_0+K(t))!} \mathrm{e}^{-\lambda t}, & j \geqslant N_0 - K(t), \\ 0, & j < N_0 - K(t), \end{cases}$$

where $N_0 = q_1 + L_p/L_c + 1$, λ *is the packet arrival rate of cross-traffic, and* $K(t)$ *is the number of packets that have left the system by* t.

(ii) *When* $t > D$,

$$\pi(t) = \pi(t \bmod D) P^{\lfloor t/D \rfloor},$$

where

$$P = \begin{pmatrix} \mathrm{e}^{\lambda D} & \lambda D \mathrm{e}^{\lambda D} & \cdots & \frac{(\lambda D)^{j-1}}{(j-1)!} \mathrm{e}^{-\lambda D} & \cdots \\ \mathrm{e}^{\lambda D} & \lambda D \mathrm{e}^{\lambda D} & \cdots & \frac{(\lambda D)^{j-1}}{(j-1)!} \mathrm{e}^{-\lambda D} & \cdots \\ 0 & \mathrm{e}^{\lambda D} & \cdots & \frac{(\lambda D)^{j-2}}{(j-2)!} \mathrm{e}^{-\lambda D} & \cdots \\ 0 & 0 & \mathrm{e}^{\lambda D} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Proof. See Appendix. □

Hence, we can obtain the state vector when the second probing packet arrives at the queue, i.e., $\pi(\varDelta_{\text{in}})$ for any arbitrary value of $\varDelta_{\text{in}}$ from Lemma 1.

Now, we derive the waiting time distribution of the second probing packet. Let the random variable $W(T)$ denote the waiting time of an arbitrary packet which arrives at an M/D/1 queue at $t = T > 0$. We will call this packet a virtual packet. Then, we have $W_2 = W(\varDelta_{\text{in}})$ with initially holding N_0 packets in the system. Hence, if we can find the distribution of $W(\varDelta_{\text{in}})$ for any given value of $\varDelta_{\text{in}}$, then it will be straightforward to obtain the relationship between $\varDelta_{\text{out}}$ and $\varDelta_{\text{in}}$ from (2). The virtual waiting time distribution $P(W(\varDelta_{\text{in}}) \leqslant kD - v)$ for all $k \in \mathbb{N}$ and $v \in (0, D]$, can be determined by concentrating on the queueing position of the virtual packet at $t = \varDelta_{\text{in}} + D - v$. If we define the random variable $N_{\varDelta_{\text{in}}}(t)$ as the number of packets arriving before $\varDelta_{\text{in}}$ that are still in the system at t, then the cumulative distribution function (CDF) of the waiting time $W(\varDelta_{\text{in}})$ is as follows:

Proposition 1. *The CDF* $F_{\varDelta_{\text{in}}}(x) = P\{W(\varDelta_{\text{in}}) \leqslant x\}$ *is*

$$F_{\varDelta_{\text{in}}}(x) = \begin{cases} \sum_{j=0}^{\lfloor \frac{x}{D} \rfloor - N_0 + K(y+D)} \frac{(\lambda \varDelta_{\text{in}})^j}{j!} \mathrm{e}^{-\lambda \varDelta_{\text{in}}}, & y \leqslant 0, \\ \sum_{j=0}^{\lfloor \frac{x}{D} \rfloor} Q_{\lfloor \frac{x}{D} \rfloor - j}(y) \frac{(\lambda z)^j}{j!} \mathrm{e}^{-\lambda z}, & \textit{otherwise}, \end{cases}$$

where $y := \varDelta_{in} - D + (x \bmod D)$, $z := D - (x \bmod D)$, *and* $Q_m(t) := \sum_{i=0}^{m+1} \pi_i(t)$.

Proof. See Appendix. □

3.3. M/D/1 delay model of packet-pair probing

By combining all these results, we derive a delay model of packet-pair probing in a single hop. First, for given value of $\varDelta_{\text{in}}$ and q_1,

$$\mathbb{E}[W_2 | q_1] = \mathbb{E}[W(\varDelta_{\text{in}}) | q_1] = \int_0^\infty (1 - F_{\varDelta_{\text{in}}}(x))\, \mathrm{d}x.$$

Here, $F_{\varDelta_{\text{in}}}(x)$ can be obtained from Proposition 1 with $N_0 = q_1 + L_p/L_c + 1$. Finally from (2),

$$\mathbb{E}[\varDelta_{\text{out}}] = \varDelta_{\text{in}} + \mathbb{E}[\mathbb{E}[W_2 | q_1]] - \mathbb{E}[W_1(q_1)]. \tag{3}$$

Note that we can calculate $\mathbb{E}[\mathbb{E}[W_2|q_1]]$ and $\mathbb{E}[W_1(q_1)]$ in (3) by using the distribution of q_1, the steady-state solution of the M/D/1 queue. An efficient algorithm to obtain the steady-state solution can be found in, for example [20].

Now, in order to enhance our understanding of the proposed model (3), consider the two extreme cases where $\varDelta_{\text{in}} \to 0$ and $\varDelta_{\text{in}} \to \infty$. First, we have the following result when $\varDelta_{\text{in}} \to 0$:

Proposition 2. *When* $\varDelta_{in} \ll 1/\lambda$, *the relationship between the input and the output probing gaps becomes*

$$\mathbb{E}[\varDelta_{\text{out}}] = \frac{r}{C} \varDelta_{\text{in}} + \frac{L_p}{C} + o(\varDelta_{\text{in}}).$$

Proof. From the basic property of the Poisson process, when $\varDelta_{\text{in}} \ll 1/\lambda$,

$$A(0, \varDelta_{\text{in}}) = \begin{cases} 1, & \text{with probability } \lambda \varDelta_{\text{in}} + o(\varDelta_{\text{in}}), \\ 0, & \text{with probability } 1 - \lambda \varDelta_{\text{in}} + o(\varDelta_{\text{in}}). \end{cases}$$

Then, for given q_1,

$$\begin{aligned} F_{\varDelta_{\text{in}}}(x) = {} & (1 - \lambda \varDelta_{\text{in}}) I(x - N_0 D) \\ & + \lambda \varDelta_{\text{in}} I(x - (N_0 + 1) D) \\ & + o(\varDelta_{\text{in}})(1 - I(x - MD)), \end{aligned}$$

where $I(\cdot)$ is the indicator function and M is a sufficiently large number independent of Δ_{in}. Hence,

$$\mathbb{E}[W_2|q_1] = \int_0^\infty (1 - F_{\Delta_{\text{in}}}(x))\,\mathrm{d}x$$
$$= \lambda D\Delta_{\text{in}} + N_0 D + \mathrm{o}(\Delta_{\text{in}}).$$

By using $W_1 = (N_0 - L_{\text{p}}/L_{\text{c}})D$,

$$\mathbb{E}[\Delta_{\text{out}}|q_1] - \Delta_{\text{in}} = \mathbb{E}[W_2|q_1] - W_1$$
$$= \lambda D\Delta_{\text{in}} + L_{\text{p}}D/L_{\text{c}} + \mathrm{o}(\Delta_{\text{in}}). \quad (4)$$

Since Δ_{out} is independent of q_1, with $D = L_{\text{c}}/C$ and $r = \lambda L_{\text{c}}$, (4) becomes $\mathbb{E}[\Delta_{\text{out}}] = \frac{r}{C}\Delta_{\text{in}} + \frac{L_{\text{p}}}{C} + \mathrm{o}(\Delta_{\text{in}})$. □

Here, we should note that the result in Proposition 2 is identical to the full-utilization case of the deterministic model in (1).

When $\Delta_{\text{in}} \to \infty$, we have the following result:

Proposition 3. *When $\Delta_{in} \to \infty$,*

$$\mathbb{E}[\Delta_{\text{out}}] \sim \Delta_{\text{in}},$$

where $f(t) \sim g(t)$ represents $f(t)/g(t) \to 1$ as $t \to \infty$.

Proof. From (2),

$$\mathbb{E}[\Delta_{\text{out}}] - \Delta_{\text{in}} = \mathbb{E}[W_2] - \mathbb{E}[W_1]$$
$$= \mathbb{E}[\mathbb{E}[W_2(q_2)]|q_1] - \mathbb{E}[W_1(q_1)].$$

As $\Delta_{\text{in}} \to \infty$, W_2 becomes independent of N_0 and consequently, $\mathbb{E}[W_2] \sim \mathbb{E}[W_1]$. Hence, when Δ_{in} is very large, $\mathbb{E}[\Delta_{\text{out}}] \sim \Delta_{\text{in}}$. □

Here, again note that the result in Proposition 3 is identical to the under-utilization case of the deterministic model in (1). Hence, from Propositions 2 and 3, the deterministic model (1) can be considered as an *asymptote* of the proposed stochastic model.

The relationship in Proposition 3 can also be explained in a qualitative manner from a physical viewpoint. The difference between $\mathbb{E}[W_2]$ and $\mathbb{E}[W_1]$ results from the first probing packet itself. The arrival of the first probing packet perturbs the M/D/1 system as it acts as an additional packet in the initial state of the system. Consequently, this perturbation makes $\mathbb{E}[W_2] \geqslant \mathbb{E}[W_1]$. The effect of the first probing packet will eventually disappear as $\Delta_{\text{in}} \to \infty$, and we have Proposition 3.

4. Stochastic packet-pair model: multi-hop case

Here, we derive a model for a multi-hop path which has a single tight link. Consider M links in an end-to-end path as in Fig. 2. Let C_i, X_i, $q_{1,i}$, and $q_{2,i}$ denote, respectively, the link capacity, the amount of cross-traffic, and the queue length of the first and the second probing packets on the ith link, $1 \leqslant i \leqslant M$. Also, the average packet sending rate of X_i and the link utilization of link i are denoted by λ_i and u_i, respectively. Furthermore, let Δ_i denote the inter-departure time from the ith queue. Then Δ_i becomes the inter-arrival time at the $(i+1)$th queue, and $\Delta_{\text{in}} = \Delta_0$ and $\Delta_{\text{out}} = \Delta_M$. With the above notations, the available bandwidth of the ith link is $A_i = C_i(1 - u_i)$ and the available bandwidth of a path is A_s where $s = \arg_i \min A_i$. The assumption of a single tight link represents the case where $A_s \ll A_i$ for $i = 1, \ldots, M$, $i \neq s$.

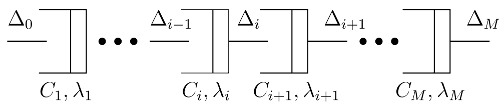


Fig. 2. Multi-hop model.

Similarly as the single-hop case in (2), we have

$$\Delta_i - \Delta_{i-1} = W_{2,i} - W_{1,i}, \quad i = 1, \ldots, M. \quad (5)$$

Hence, by adding all the terms on each side from $i = 1$ to M, we have

$$\Delta_{\text{out}} - \Delta_{\text{in}} = \Delta_M - \Delta_0 = \sum_{i=1}^{M}(W_{2,i} - W_{1,i}).$$

Then,

$$\mathbb{E}[\Delta_{\text{out}}] = \Delta_{\text{in}} + \sum_{i=1}^{M}\mathbb{E}[W_{2,i} - W_{1,i}]$$
$$= \Delta_{\text{in}} + \sum_{i=1}^{M}\mathbb{E}[\mathbb{E}[W_{2,i} - W_{1,i}|q_{1,i}]]. \quad (6)$$

Now, we first consider a two-hop case and extend the result to the general multi-hop path.

4.1. Two-hop case

Before dealing with a general multi-hop path, we first consider a two-hop path, i.e., $M = 2$ in Fig. 2. First, we assume that $\Delta_1^* < \Delta_2^*$. This implies that the available bandwidth of the path is that of the second link. From (6) with $M = 2$, we have

$$\mathbb{E}[\Delta_{\text{out}}] = \Delta_{\text{in}} + \mathbb{E}[W_{2,1} - W_{1,1}] + \mathbb{E}[W_{2,2} - W_{1,2}].$$

Now, we derive an explicit relationship between $\mathbb{E}[\Delta_{\text{out}}]$ and Δ_{in} under the assumption of a single tight link. If we further assume that $\Delta_1^* \ll \Delta_2^*$ and

Δ_{in} is around Δ_2^*, then $\Delta_{\text{in}} \gg \Delta_1^*$. Hence, we have $\mathbb{E}[W_{2,1} - W_{1,1}] \approx 0$ from Proposition 3. In this case, we have

$$\mathbb{E}[\Delta_{\text{out}}] \approx \Delta_{\text{in}} + \mathbb{E}[W_{2,2} - W_{1,2}]. \tag{7}$$

When Δ_0 is around Δ_1^*, $\Delta_{\text{in}} \ll \Delta_2^*$ and

$$\mathbb{E}[\Delta_1] \approx \Delta_{\text{in}} + \mathbb{E}[W_{2,1} - W_{1,1}].$$

Also,

$$\mathbb{E}[\Delta_{\text{out}}] \approx u_2 \mathbb{E}[\Delta_1] + \frac{L_{\text{p}}}{C_2}.$$

Hence,

$$\mathbb{E}[\Delta_{\text{out}}] \approx u_2(\Delta_{\text{in}} + \mathbb{E}[W_{2,1} - W_{1,1}]) + \frac{L_{\text{p}}}{C_2}. \tag{8}$$

Both (7) and (8) can be solved in the same manner as the single-link case. Note that, in finding the available bandwidth along the path, only (7) is needed. Also, note that (7) and (8) are valid under the assumption of $\Delta_1^* \ll \Delta_2^*$. As Δ_1^* approaches Δ_2^*, i.e., the available bandwidth of the first link approaches that of the second link, the assumption is no longer valid, and (7) and (8) become inaccurate.

Now, let us consider the case of $\Delta_1^* > \Delta_2^*$, i.e., the available bandwidth of the path is that of the first link. Similarly to the case of $\Delta_1^* < \Delta_2^*$, we further assume $\Delta_1^* \gg \Delta_2^*$. When Δ_{in} is around Δ_1^*, similarly as in (7) we have

$$\mathbb{E}[\Delta_{\text{out}}] \approx \Delta_{\text{in}} + \mathbb{E}[W_{2,1} - W_{1,1}]. \tag{9}$$

When Δ_{in} is around Δ_2^*,

$$\mathbb{E}[\Delta_1] \approx \Delta_{\text{in}} + \mathbb{E}[W_{2,1} - W_{1,1}].$$

we also have $\mathbb{E}[\Delta_{\text{out}}] \approx \mathbb{E}[\Delta_1]$ with $\Delta_1^* \gg \Delta_2^*$. Hence,

$$\mathbb{E}[\Delta_{\text{out}}] \approx \Delta_{\text{in}} + \mathbb{E}[W_{2,1} - W_{1,1}]. \tag{10}$$

From (9) and (10), we know that the characteristic value of the second link, i.e., Δ_2^* is *not observable* via the packet-pair technique when the available bandwidth of the first link is smaller than that of the second link. We will also verify this phenomenon in the simulations in Section 5.

4.2. Multi-hop case

By extending the two-hop analysis to the M-hop case,

$$\begin{aligned}\mathbb{E}[\Delta_{\text{out}}] &= \Delta_{\text{in}} + \sum_{i=1}^{M} \mathbb{E}[W_{2,i} - W_{1,i}] \\ &\approx \Delta_{\text{in}} + \sum_{i=1}^{s} \mathbb{E}[W_{2,i} - W_{1,i}],\end{aligned}$$

where the sth link has the minimum available bandwidth. Hence, based on the packet-pair technique, we cannot observe the characteristic value of any link after the tight link. In other words, any link after the tight link does not affect the relationship between the input and the output probing gaps. Consequently, the available bandwidth of any link after the tight link is *not observable* in an end-to-end manner if we use the packet-pair technique. We can obtain the information only on the available bandwidth of the ith link where $i \leqslant s$.

If we are concerned only with obtaining the available bandwidth of an end-to-end path, it is sufficient to observe the behavior of the packet-pair technique in the vicinity of $\Delta_{\text{in}} = \Delta_s^*$. If we assume that $\Delta_s^* \gg \Delta_i^*$, $i = 1, \ldots, M$, $i \neq s$, i.e., there is a single tight link as in the cases of (7) and (9), we can derive the following approximate model for the multi-hop path:

$$\begin{aligned}\mathbb{E}[\Delta_{\text{out}}] &= \Delta_{\text{in}} + \sum_{i=1}^{M} \mathbb{E}[W_{2,i} - W_{1,i}] \\ &\approx \Delta_{\text{in}} + \mathbb{E}[W_{2,s} - W_{1,s}].\end{aligned} \tag{11}$$

The multi-hop model (11) shows that we can obtain the available bandwidth of the path from the local analysis which is identical to the single-hop case in (3) when we assume a single tight link.

4.3. Discussion on the proposed model

Here, we discuss the impact of the proposed packet-pair model on available bandwidth estimation. Since the multi-hop analysis is identical to the single-hop case when we are concerned only about the available bandwidth of the path under the assumption of a single tight link, we discuss the single-hop model (3) in relation to the available bandwidth estimation. We show the difference between the proposed model (3) and the deterministic model (1) in Fig. 3. The figure shows that available bandwidth estimation algorithms based on (1) may give an inaccurate estimate of the characteristic value Δ^* because of the modeling error. Further, the error is most significant when $\Delta_{\text{in}} = \Delta^*$ (24 ms in Fig. 3). This error between the deterministic and the stochastic models has been also identified and termed as the probing bias in [14]. The influence of the probing bias on available bandwidth measurement is significant since the probing bias makes it difficult to estimate the characteristic value by observing the relationship between Δ_{out} and Δ_{in}.

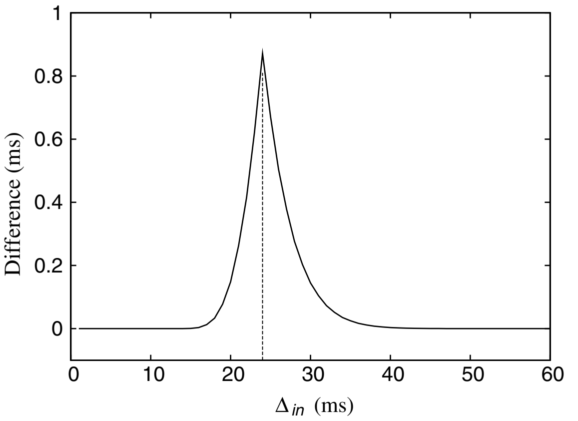


Fig. 3. Difference between the proposed and the deterministic models.

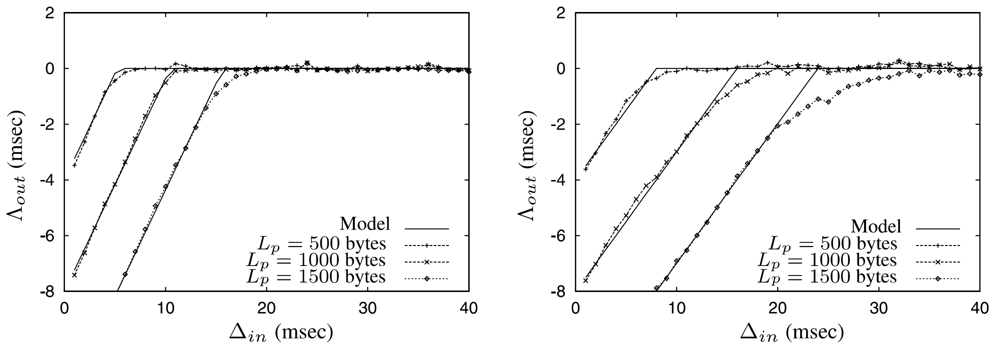
5. Model validation

In this section, we validate our stochastic model via ns-2 simulations and experimental results.

5.1. ns-2 simulations for single-hop topology

We perform simulations for a single-hop topology composed of two drop-tail routers that are connected via a link with the capacity of 1 Mb/s and a propagation delay of 15 ms. First, we investigate the deterministic model (1). In the simulation, the cross traffic consists of N Pareto sources, and each source generates packets at the rate of 32 Kb/s with the packet size of $L_c = 50$ bytes. Fig. 4(a) and (b) show $\Delta_{in} - \overline{\Delta}_{out}$ versus Δ_{in} for $N = 7$ and 15, respectively, where $\overline{\Delta}_{out} := \mathbb{E}[\Delta_{out}]$. The sizes of probing packets are 500, 1000, and 1500 bytes, respectively. Note that we introduce $\Delta_{in} - \overline{\Delta}_{out} =: \Lambda_{out}$ to highlight the change of $\frac{\partial \overline{\Delta}_{out}}{\partial \Delta_{in}}$ around the characteristic value $\Delta^* \left(= \frac{L_p + q}{C - r}\right)$. Each data point is an average value of 300 trials. The solid lines are obtained from the deterministic model in (1) with $q = 0$. We can see that the simulation data asymptotically converges to the analytical values. However, the modeling error around $\Delta_{in} = \Delta^*$ becomes larger as the link utilization or the probing packet size increases. $\frac{\partial \Delta_{out}}{\partial \Delta_{in}}$ changes smoothly around $\Delta_{in} = \Delta^*$ in the simulation results while it changes abruptly at $\Delta_{in} = \Delta^*$ in the deterministic model. This implies that the stochastic nature of cross-traffic makes it difficult to find the value of Δ^* in practice.

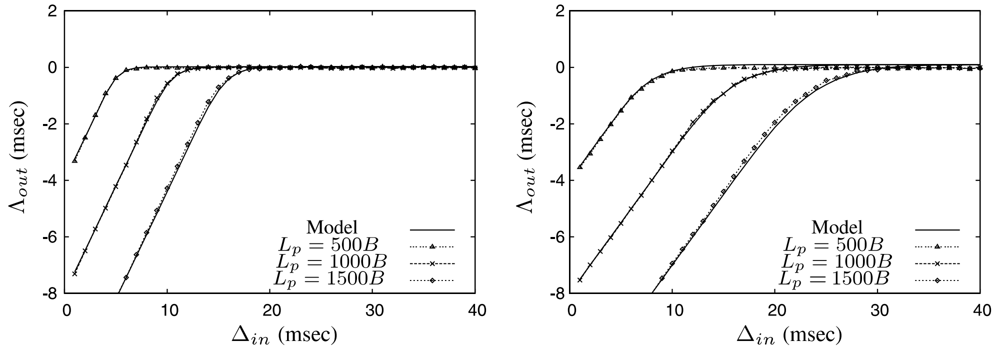
Now, we validate the stochastic model in (3). Figs. 5–7 show Λ_{out} versus Δ_{in} for Poisson, exponential ON–OFF, and Pareto ON–OFF cross-traffic, respectively. Even though we have assumed Poisson cross-traffic in our analysis, we perform simulations with exponential ON–OFF and Pareto ON–OFF cross-traffic to see the effect of the cross-traffic distribution. Each figure shows the result for different sizes of probing packets ($L_p = 500$, 1000, and 1500 bytes) with the utilization of 24% and 50%, respectively. In all the simulations, we use $L_c = 50$ bytes for the packet size of cross-traffic. In Fig. 5, we can see that each of the simulation results agrees very well with the stochastic model in (3). This is because the Poisson cross-traffic is used both in the model and the simulations. Now, we consider the exponential ON–OFF sources in Fig. 6. In Fig. 6, the Poisson cross-traffic is used for the model as in Fig. 5 while the exponential ON–OFF cross traffic is employed in the simulation. Accordingly, we can see that there exist some errors between the model and the simulation data. Note that the slope of Λ_{out} changes more slowly around $\Delta_{in} = \Delta^*$ in the simulation data than in the model. Next, we consider the case of the Pareto ON–OFF cross-traf-



(a) Utilization = 24%

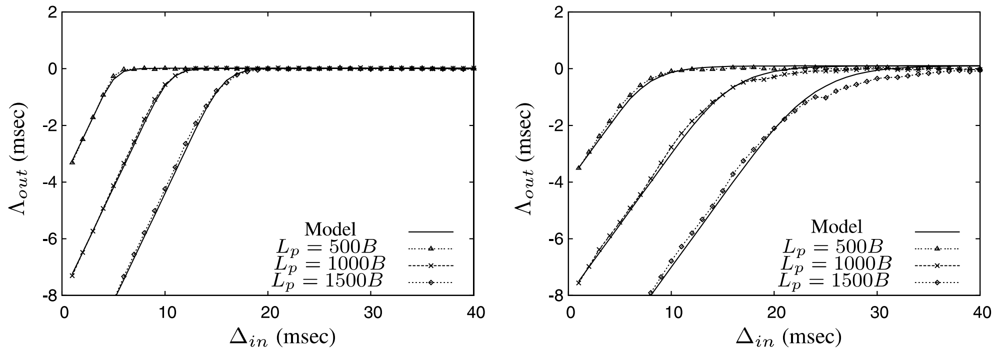
(b) Utilization = 50%

Fig. 4. Validation of the deterministic model: $\Delta_{in} - \overline{\Delta}_{out} (=: \Lambda_{out})$ versus Δ_{in}.



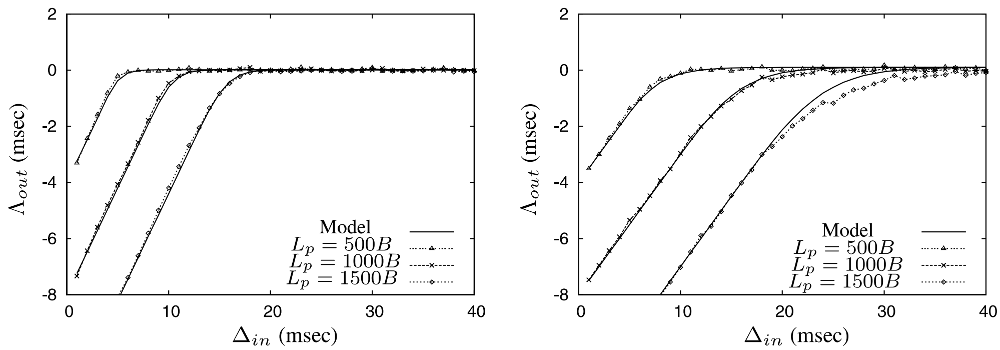
(a) Utilization = 24% (b) Utilization = 50%

Fig. 5. Λ_{out} versus Δ_{in} for the Poisson cross traffic in a single-hop topology.



(a) Utilization = 24% (b) Utilization = 50%

Fig. 6. Λ_{out} versus Δ_{in} for the exponential ON–OFF cross-traffic in a single-hop topology.



(a) Utilization = 24% (b) Utilization = 50%

Fig. 7. Λ_{out} versus Δ_{in} for the Pareto ON–OFF cross-traffic in a single-hop topology.

fic in Fig. 7. Similarly as in Figs. 6 and 7 show some error around $\Delta_{\text{in}} = \Delta^*$ for the same reason. Note that the variance of the Pareto ON–OFF traffic is even larger than that of the exponential ON–OFF traffic, and Λ_{out} changes more slowly in the simulation data of Fig. 7 than that of Fig. 6.

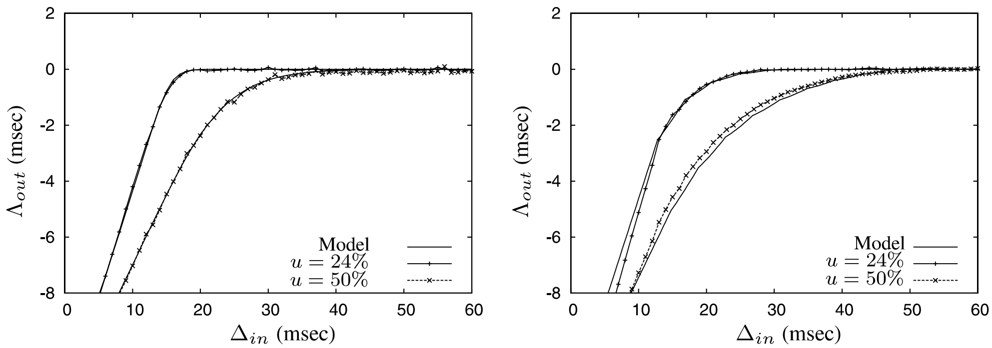
Now, we vary the packet size of the Poisson cross-traffic to investigate the effect of the packet size on the model accuracy. Fig. 8(a) and (b) show the simulation results of Λ_{out} vs. Δ_{in} under different link utilizations for the packet sizes of 50 and 500 bytes, respectively. When the packet size is 50 bytes, we can verify from Fig. 8(a) that the proposed model is quite accurate. When the packet size is 500 bytes in Fig. 8(b), the proposed model still agrees quite well with the simulation result. Note that each point in Fig. 8 is an average value of 30,000 trials. We will also investigate the effect of the packet size in cross traffic with the empirical results in Section 5.3.

5.2. ns-2 simulations for two-hop topology

We now validate the stochastic model for the two-hop topology, which is composed of three drop-tail routers that are connected via two links with capacity of either $C_1 = 2$ Mb/s and $C_2 = 1$ Mb/s (which corresponds to the case $\Delta_1^* < \Delta_2^*$) or $C_1 = 1$ Mb/s and $C_2 = 2$ Mb/s (which corresponds to the case $\Delta_1^* > \Delta_2^*$). The cross-traffic on the first link (the second link) is composed of 23 (15) Pareto sources that give an aggregate rate of $r = 736$ Kb/s ($r = 480$ Kb/s). The packet size L_c of cross-traffic is 100 bytes. Fig. 9(a) and (b) show the analytical and simulation results for the cases of $\Delta_1^* < \Delta_2^*$ and $\Delta_1^* > \Delta_2^*$, respectively. In the case of $\Delta_1^* < \Delta_2^*$ ($L_p = 1500$ bytes, $C_1 = 2$ Mb/s, $C_2 = 1$ Mb/s), we can see two changes in $\frac{\partial \Lambda_{out}}{\partial \Delta_{in}}$ around $\Delta_{in} = \Delta_1^* = 9.5$ ms and $\Delta_{in} = \Delta_2^* = 23.1$ ms in Fig. 9. Note that Δ_2^* gives the information on the available bandwidth along the path. In the case of $\Delta_1^* > \Delta_2^*$ ($L_p = 1500$ bytes, $C_1 = 1$ Mb/s, $C_2 = 2$ Mb/s), we only see one change in $\frac{\partial \Lambda_{out}}{\partial \Delta_{in}}$ around $\Delta_{in} = \Delta_1^* = 23.1$ ms. Hence, Fig. 9 shows that the multi-hop model in (11) agrees quite well around the largest characteristic value, which gives information on the available bandwidth of the path.

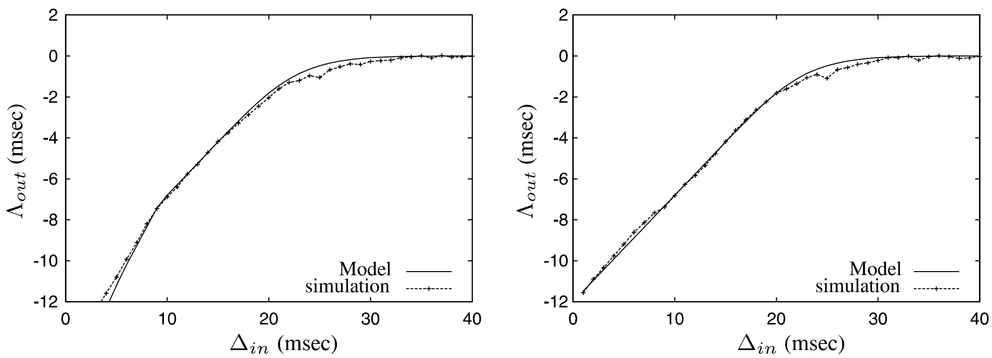
5.3. Empirical results in two-hop network

Here, we investigate the proposed stochastic model in a real network environment. The network is composed of three routers R_1, R_2, and R_3 that are



(a) $L_c = 100$ bytes (b) $L_c = 500$ bytes

Fig. 8. Packet size effect of cross-traffic ($L_p = 1500$ bytes).



(a) When $\Delta_1^* < \Delta_2^*$: Both Δ_1^* and Δ_2^* are observable (b) When $\Delta_1^* > \Delta_2^*$: Δ_2^* is not observable

Fig. 9. Λ_{out} versus Δ_{in} for the Pareto ON–OFF cross-traffic in the two-hop topology.

connected via two 100 Mb/s links and five hosts, denoted by l_1, l_2, and H_i, $i = 1, \ldots, 5$, respectively, as depicted in Fig. 10. Here, each host is a PC with a single 2.66 GHz processor and 1 GByte RAM that runs Redhat Linux Release 9.0 with the kernel version 2.4.20–31.9. The routers used in the experiments are Cisco 1700 Series. The path for which we injected the probing packets was $\mathscr{P}: H_1 \to R_1 \to R_2 \to R_3 \to H_3$, which constitutes a two-hop path. Each of the experimental data points in Figs. 11 and 12 is an average of 5 trials.

The Poisson cross-traffic is generated and traverses the path from H_5 to H_4. Hence, the link $l_2: R_2 \to R_3$ becomes the tight link of the path $\mathscr{P}$. Fig. 11(a) and (b) show Λ_{out} for $L_p = 500$, 1000, and 1500 bytes, when L_c is 250 bytes and the link utilization is 25% and 50%, respectively. We can see that the stochastic model agrees quite well with the experimental data in the figure. The path $\mathscr{P}$ used in the experiments was composed of two links $l_1: R_1 \to R_2$ and $l_2: R_2 \to R_3$, and the corresponding characteristic values of the link l_1 are

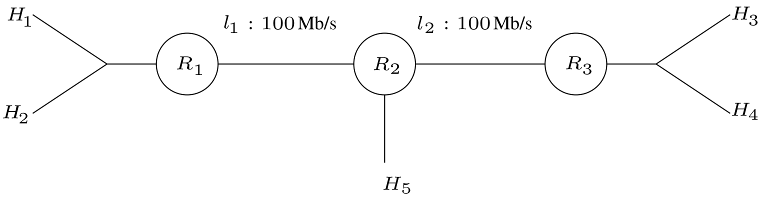


Fig. 10. Topology used in the experiments.

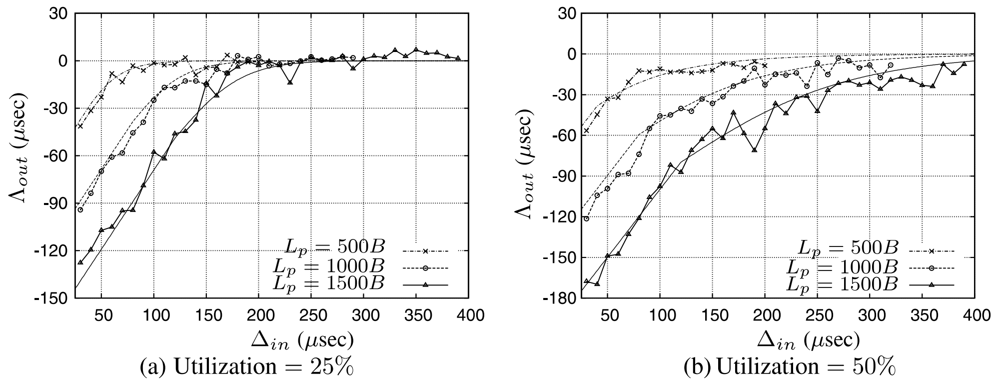


Fig. 11. Λ_{out} versus Δ_{in} with the Poisson cross traffic of $L_c = 250$ bytes in the two-hop experiment.

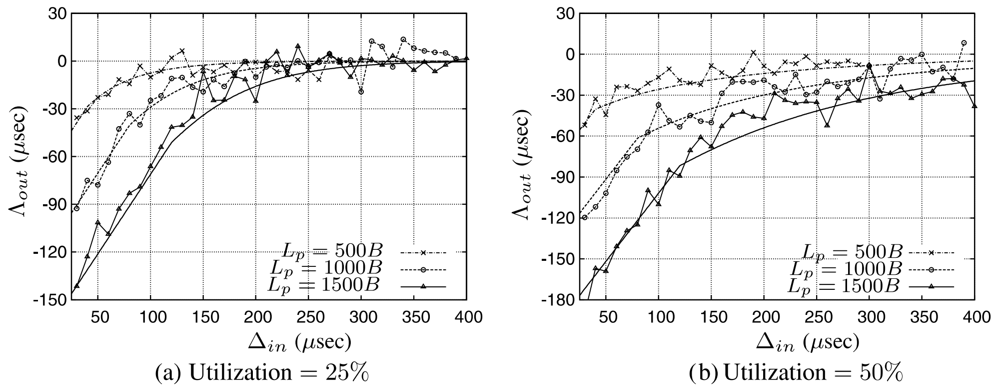


Fig. 12. Λ_{out} versus Δ_{in} with the Poisson cross-traffic of $L_c = 750$ bytes in the two-hop experiment.

$\Delta_1^* = L_p/C_1 = 40$, 80, and 120 μs for $L_p = 500$, 1000, and 1500 bytes, respectively. Similarly, the characteristic values of link l_2 when $u = 25\%$ ($u = 50\%$) are $\Delta_2^* = L_p/(C_2 - r_2) = 160/3$, 320/3, and 160 μs (80, 160, and 240 μs) for $L_p = 500$, 1000, and 1500 bytes, respectively. Since $\Delta_1^* < \Delta_2^*$, we can observe both Δ_1^* and Δ_2^* in Figs. 11 and 12 as in Fig. 9(a). Note that the experimental data changes smoothly around $\Delta_{in} = \Delta_2^*$ as expected, and this makes it difficult to find Δ_2^*. This difficulty is not apparent with the deterministic model in (1). Fig. 12 shows the results of the similar experiments with $L_c = 750$ bytes. From the figure, we can see that the stochastic model follows the experimental data quite well. In summary, the experimental results show that the proposed model is quite accurate in a real environment even when the packet size of cross-traffic is large ($L_c = 250$ bytes in Fig. 11 and $L_c = 750$ bytes in Fig. 12).

6. Related work

The packet-pair technique originally appeared in the seminal work by Jacobson [3], Keshav [4], and Bolot [5]. This early work was followed by extensive research into the packet-pair technique. Here, we investigate several recent models of the packet-pair technique for bandwidth measurement.

In [7], Dovrolis et al. proposed the following relationship between the input and the output probing gaps:

$$\Delta_i = \begin{cases} \tau_i + d_i^2, & \Delta_{i-1} \leqslant \tau_i + d_i^1, \\ \Delta_{i-1} + (d_i^2 - d_i^1), & \text{otherwise}, \end{cases} \tag{12}$$

where τ_i and d_i^1 denote the transmission delay and the queueing delay of the first probing packet at the ith link, respectively. Also, d_i^2 denote any additional queueing delay of the second probing packet at the ith link after the first probing packet has departed from the link. Hence, $d_i^2 = W_{2,i}$ when $\Delta_{i-1} \geqslant \tau_i + d_i^1$, but $d_i^2 \neq W_{2,i}$ when $\Delta_{i-1} \leqslant \tau_i + d_i^1$. Under the fluid cross traffic with a constant rate, we have $\tau_i = L_p/C$, $d_i^1 = r\Delta_{i-1}$ when $\Delta_{i-1} \leqslant \frac{L_p}{C_i} + d_i^1$ and $d_i^1 = d_i^2$ when $\Delta_{i-1} > \frac{L_p}{C_i} + d_i^1$. Hence, (12) exactly matches the deterministic model (1) under the assumption of fluid cross-traffic. Note that (12) is more general than the deterministic model (1) since (12) describes the stochastic relationship between Δ_{i-1} and Δ_i. We can easily show that (12) can be transformed into (5) as follows: from the definitions, we can easily know that $d_i^1 = W_{1,i}$ and $\tau_i + d_i^1 + d_i^2 = \Delta_{i-1} + W_{2,i}$ when $\Delta_{i-1} \leqslant \tau_i + d_i^1$. When $\Delta_{i-1} \geqslant \tau_i + d_i^1$, we have $d_i^1 = W_{1,i}$ and $d_i^2 = W_{2,i}$. Hence, with these relationships, (12) can be converted into (5).

As already explained, a deterministic packet-pair model was derived under an assumption of fluid cross-traffic in [8], which is a deterministic version of (12). As we have shown in Propositions 2 and 3, this deterministic model corresponds to an asymptote of the proposed M/D/1 model.

More recently, a stochastic analysis of packet pair/train has been given in [14], in which the following upper and lower bounds for the single-hop case was derived based on a sample-path analysis

$$L(\mathbb{E}[\Delta_{out}]) = \begin{cases} \frac{r}{C}\Delta_{in} + \frac{L_p}{C}, & \Delta_{in} \leqslant \frac{L_p}{C-r}, \\ \Delta_{in}, & \text{otherwise}, \end{cases} \tag{13}$$

$$U(\mathbb{E}[\Delta_{out}]) = \begin{cases} \frac{r}{C}\Delta_{in} + \frac{L_p}{C}, & \Delta_{in} \leqslant \frac{L_p}{C}, \\ \frac{r}{C}\Delta_{in} + \Delta_{in}, & \frac{L_p}{C} \leqslant \Delta_{in} \leqslant \frac{L_p}{r}, \\ \Delta_{in} + \frac{L_p}{C}, & \text{otherwise}. \end{cases} \tag{14}$$

We can verify that (13) is identical to the deterministic model (1), which is an asymptote of the stochastic model (3).

Furthermore, from (14), we have the following lower bound for $\Lambda_{out} = \Delta_{in} - \mathbb{E}[\Delta_{out}]$

$$L(\Lambda_{out}) = \begin{cases} (1 - \frac{r}{C})\Delta_{in} - \frac{L_p}{C}, & \Delta_{in} \leqslant \frac{L_p}{C}, \\ -\frac{r}{C}\Delta_{in}, & \frac{L_p}{C} \leqslant \Delta_{in} \leqslant \frac{L_p}{r}, \\ -\frac{L_p}{C}, & \text{otherwise}. \end{cases} \tag{15}$$

Since $\mathbb{E}[\mathbb{E}[W_2|q_1]]$ in (3) is a decreasing function of Δ_{in}, Λ_{out} is an increasing function of Δ_{in}. Hence, we know from (15) that the upper bound in (14) is not very tight when $\frac{L_p}{C} \leqslant \Delta_{in} \leqslant \frac{L_p}{r}$. Overall, the sample-path analysis in [14] has successfully shown the general characteristics of the packet pair/train probing technique. Our analysis differs from the result of [14] in that we have derived an explicit relationship under Poisson cross-traffic. Further, we have given a novel insight that the transient queueing analysis can accurately describe the behavior of a packet pair. This insight has a significant importance since it provides an effective way of modeling the packet-pair technique by using queueing theory.

7. Conclusion

In this paper, we have derived an explicit packet-pair model, which reflects the stochastic nature of cross-traffic. We have investigated the mathematical relationship between the input and the output probing gaps of a packet pair in the single-hop and the multi-hop cases under the assumption of stationary Poisson cross-traffic. We have shown that the proposed model agrees very well with the ns-2 simulations and the empirical results. Based on the analysis, we have pointed out that most of the recent models of the packet-pair technique can be regarded as special cases of the proposed model. We expect that the proposed model will play an important role in developing a measurement mechanism for estimating network bandwidth.

There remain several issues for future research work. In the analysis of developing a multi-hop model, we have made an assumption of a single tight link. However, this assumption will be unrealistic if two or more links have nearly the same available bandwidth. Hence, development of a multi-hop model without this assumption remains as future work. It is also necessary to carry out large-scale measurement on the Internet in order to verify the stochastic model with real network traffic. Extension of the M/D/1 model to the $M^{[X]}/D/1$ case is another future task to take account of the multi-modal distribution of the Internet traffic packet size.

Acknowledgements

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Appendix A

A.1. Proof of Lemma 1

Here, the proof is a slight modification of the result in [17]. When $t \in (0, D]$ where $D = L_c/C$, the only packets that may have left the system since $t = 0$ are those already in service at $t = 0$. There are initially $N_0 = q_1 + L_p/L_c + 1$ packets at $t = 0$ and let $K(t)$ denote the number of packets that have left the system by the time t. With the residual service time d,

$$K(t) = \begin{cases} 1, & \text{for } d \leqslant t \leqslant D \\ 0, & \text{otherwise.} \end{cases}$$

Note that $d = 0$ and $K(t) = 1$, $t \in (0, D]$ if there is no packet in service at $t = 0$. During the time interval $(0, t]$ there will be i new arrivals with probability $\frac{(\lambda t)^i}{i!} e^{-\lambda t}$. In order to have j customers in the system at time t, we need $j - N_0 + K(t)$ new arrivals during $(0, t]$, and consequently, for any $t \in (0, D]$,

$$\pi_j(t) = \begin{cases} \frac{(\lambda t)^{j-N_0+K(t)}}{(j-N_0+K(t))!} e^{-\lambda t}, & j \geqslant N_0 - K(t), \\ 0, & j < N_0 - K(t). \end{cases}$$

When $t > D$, by conditioning on the number of packets at time t,

$$\begin{aligned} \pi_j(t+D) &= P(N(t)=0)P(j \text{ arrivals}|N(t)=0) \\ &\quad + \sum_{i=1}^{j+1} P(N(t)=i)P(j+1-i \text{ arrivals}|N(t)=i) \\ &= \pi_0(t)\frac{(\lambda D)^j}{j!}e^{-\lambda D} + \sum_{i=1}^{j+1}\pi_i(t)\frac{(\lambda D)^{j+1-i}}{(j+1-i)!}e^{-\lambda D} \end{aligned}$$

for all $j \in \mathbb{N}^0$. Let us define the transition matrix $P = [p_{ij}]$ as follows:

$$p_{ij} = \begin{cases} \frac{(\lambda D)^{j-1}}{(j-1)!} e^{-\lambda D}, & \text{for } i = 1, \\ \frac{(\lambda D)^{j-i+1}}{(j-i+1)!} e^{-\lambda D}, & \text{for } 2 \leqslant i \leqslant j+1, \\ 0, & \text{otherwise.} \end{cases}$$

Then,

$$\pi(t + D) = \pi(t)P. \tag{A.1}$$

By applying (A.1) iteratively, we can determine $\pi(t + mD)$ for any $m \in \mathbb{N}$ as follows:

$$\pi(t + mD) = \pi(t)P^m. \tag{A.2}$$

From (A.2), the following relation is obtained:

$$\pi(t) = \pi(t \bmod D)P^{\lfloor t/D \rfloor}.$$

A.2. Proof of Proposition 1

Here, the proof adopts the result in [17]. Let $N(t)$ denotes the number of packets in the system at time t. Since the arrival of the second probing packet is independent of any packet arrivals of cross traffic,

$$\begin{aligned} &P(N(t) = i| \text{ second probing packet arrives at } \Delta_{\text{in}}) \\ &\quad = \pi_i(t), \end{aligned}$$

for $0 \leqslant t < \Delta_{\text{in}}$. Since $\Delta_{\text{in}} > 0$ and $0 < v \leqslant D$, epoch $\Delta_{\text{in}} + D - v$ must take place after $t = 0$. Now we divide the problem into the following two cases: $\Delta_{\text{in}} - v \leqslant 0$ as Case 1 and $\Delta_{\text{in}} - v > 0$ as Case 2. In Case 1, $\Delta_{\text{in}} + D - v \in (0, D]$. Therefore, at $t = \Delta_{\text{in}} + D - v$, the system contains $N_0 - K(\Delta_{\text{in}} + D - v)$

packets that were already in the system at $t = 0$. Another group of packets arriving before the second probing packet are those which arrive in $(0, \Delta_{in})$. Let $A(t_1, t_2)$ denote the number of packet arrivals in interval (t_1, t_2), then

$$N_{\Delta_{in}}(\Delta_{in} + D - v) = N_0 - K(\Delta_{in} + D - v) + A(0, \Delta_{in}), t - v \leqslant 0. \quad (A.3)$$

In addition, we can obtain a relation between $W(\Delta_{in})$ and $N_{\Delta_{in}}(t)$ in the following way. Consider the two cases: $N_{\Delta_{in}}(\Delta_{in} + D - v) < k$ and $N_{\Delta_{in}}(\Delta_{in} + D - v) \geqslant k$. First, when $N_{\Delta_{in}}(\Delta_{in} + D - v) < k$, $(k-1)D$ time units later we must have $N_{\Delta_{in}}(\Delta_{in} + kD - v) < k - (k-1) = 1$. Consequently, we know that the virtual packet will not be in the queue at $t = \Delta_{in} + kD - v$. Hence,

$$N_{\Delta_{in}}(\Delta_{in} + D - v) < k \Rightarrow W(\Delta_{in}) \leqslant kD - v. \quad (A.4)$$

On the other hand, if $N_{\Delta_{in}}(\Delta_{in} + D - v) \geqslant k$, the virtual packet will remain in the queue at $t = \Delta_{in} + kD - v$. Consequently,

$$N_{\Delta_{in}}(\Delta_{in} + D - v) \geqslant k \Rightarrow W(\Delta_{in}) > kD - v. \quad (A.5)$$

By considering (A.4) and (A.5) together,

$$N_{\Delta_{in}}(\Delta_{in} + D - v) < k \Longleftrightarrow W(\Delta_{in}) \leqslant kD - v. \quad (A.6)$$

In terms of probabilities, (A.6) becomes

$$P[W(\Delta_{in}) \leqslant kD - v] = P[N_{\Delta_{in}}(\Delta_{in} + D - v) < k]. \quad (A.7)$$

From (A.3) and (A.7), when $\Delta_{in} - v \leqslant 0$,

$$P(W(\Delta_{in}) \leqslant kD - v) = P(A(0, \Delta_{in}) < k - N_0 + K(\Delta_{in} + D - v)).$$

Since $A(0, \Delta_{in})$ is Poisson, independent of N_0 and $K(\Delta_{in} + D - v)$,

$$P(W(\Delta_{in}) \leqslant kD - v) = \sum_{j=0}^{k-N_0+K(\Delta_{in}+D-v)-1} \frac{(\lambda\Delta_{in})^j}{j!} e^{-\lambda\Delta_{in}},$$

when $\Delta_{in} - v \leqslant 0$.

When $\Delta_{in} - v > 0$ in Case 2, we have $\Delta_{in} + D - v > D$. Hence, we use the fact that all packets that were in service at $t = \Delta_{in} - v$ will have left the system at $t = \Delta_{in} + D - v$, whereas all packets that were waiting for service at $t = \Delta_{in} - v$ will still be in the system at $t = \Delta_{in} + D - v$. Let $L_q(t)$ denote the queue length at time t, then at $t = \Delta_{in} + D - v$ there will be exactly $L_q(\Delta_{in} - v) + A(\Delta_{in} - v, \Delta_{in})$ packets in the system that arrived before the second probing packet. Hence, when $\Delta_{in} - v > 0$, we have

$$N_{\Delta_{in}}(\Delta_{in} + D - v) = L_q(\Delta_{in} - v) + A(\Delta_{in} - v, \Delta_{in}).$$

By conditioning on the number of arrivals during $[\Delta_{in} - v, \Delta_{in}]$, we get

$$P(W(\Delta_{in}) \leqslant kD - v | A(\Delta_{in} - v, \Delta_{in}) = j) = P(L_q(\Delta_{in} - v) < k - j),$$

when $\Delta_{in} - v > 0$. Since $A(\Delta_{in} - v, \Delta_{in})$ is independent of $L_q(\Delta_{in} - v)$,

$$P(W(\Delta_{in}) \leqslant kD - v) = \sum_{j=0}^{k-1} P(L_q(\Delta_{in} - v) < k - j) \frac{(\lambda v)^j}{j!} e^{-\lambda v},$$

when $\Delta_{in} - v > 0$. By using the cumulative probability $Q_m(t) = \sum_{i=0}^{m+1} \pi_i(t)$,

$$P(W(\Delta_{in}) \leqslant kD - v) = \sum_{j=0}^{k-1} Q_{k-j-1}(\Delta_{in} - v) \frac{(\lambda v)^j}{j!} e^{-\lambda v}. \quad (A.8)$$

Note that $Q_{k-j-1}(\Delta_{in} - v)$ can be easily calculated from Lemma 1. By substituting $x = kD - v$ and $k = \lfloor \frac{x}{D} \rfloor + 1$ in (A.8), the waiting time distribution $F_{\Delta_{in}}(x) = P(W(\Delta_{in}) \leqslant x)$ is obtained as follows:

$$F_{\Delta_{in}}(x) = \begin{cases} \sum_{j=0}^{\lfloor \frac{x}{D} \rfloor - N_0 + K(y+D)} \frac{(\lambda\Delta_{in})^j}{j!} e^{-\lambda\Delta_{in}}, & y \leqslant 0, \\ \sum_{j=0}^{\lfloor \frac{x}{D} \rfloor} Q_{\lfloor \frac{x}{D} \rfloor - j}(y) \frac{(\lambda z)^j}{j!} e^{-\lambda z}, & \text{otherwise}, \end{cases}$$

where $y := \Delta_{in} - D + (x \bmod D)$ and $z := D - (x \bmod D)$.

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