Capacity Analysis of Best-Effort Broadcasting Services with Reliability Constraint

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Abstract—In this paper, we consider best-effort broadcast service (BEBS) in wireless cellular networks, which can be used for location-based advertising or real-time multimedia broadcasting. Contrary to the conventional broadcasting service that aims to assure reliable data delivery to all users, we establish the objective of BEBS to maximize the effective capacity of wireless resource at the cost of reliability. In this context, we introduce the notion of target reliability and formulate the problem of maximizing the capacity of BEBS under a constraint of reliability. We derive the analysis model to investigate the effect of target reliability on the achievable capacity and validate the analysis model by comparing the analysis results with the simulation results. Both analysis and simulation results confirm that the BEBS significantly improves the effective capacity by relaxing the reliability constraint, compared to the conventional reliable broadcasting.

Index Terms—Broadcasting, capacity, reliability, cellular networks

I. INTRODUCTION

Recently, multicasting and broadcasting services come into the spotlight as emerging applications in the next generation wireless cellular network. They can be used for location-based mobile advertising, news/information broadcast, mobile IPTV, emergency alert, and so on. To support these services, WiMAX forum and 3GPP has standardized Multicast and Broadcast Service (MBS) and enhanced Multimedia Broadcast and Multicast Service (eMBMS), respectively [1], [2]. Compared to the unicast, multicast/broadcast improves the wireless spectrum efficiency, since multiple users can concurrently receive the identical data transmitted by a base station (BS). Meanwhile, the spectral efficiency can be further enhanced by exploiting multiple modulation and coding schemes (MCSs); BS transmits data with an appropriate MCS, based on the channel state of the user, which is referred to as adaptive modulation and coding (AMC) scheme. However, it is a challenging task to apply AMC to multicast/broadcast service, because the channel state may be significantly different from each other user. A conventional approach to the multicast/broadcast is to transmit data with the most robust MCS, such that even a user in the worst channel condition can decode the data successfully. Although this approach can assure reliable data delivery to all the users, it does not fully take advantage of AMC to improve the spectral efficiency. Existing studies in the literature of multicast/broadcast mostly focused on the reliable and efficient data delivery and/or on the effective feedback mechanism [3]–[6].

In this study, we consider best-effort broadcasting service (BEBS). Contrary to the conventional reliable multicast/broadcast service, the objective of BEBS is to maximize the capacity of wireless channel at the cost of reliability. Our work is motivated by the intuition that the channel efficiency can be improved by relaxing the reliability and there exists a fundamental tradeoff between reliability and efficiency as follows. Consider that the broadcasting data is transmitted with an aggressive MCS. Then, the amount of channel resource required to support the BEBS decreases due to the increase of transmission rate, improving the channel efficiency. On the other hand, the reliability can be degraded since the users in the poor channel condition cannot successfully receive the data due to the increase of bit error rate. In this context, we introduce the notion of target reliability and formulate the problem of BEBS as the problem of maximizing channel capacity under the constraint of reliability. Then, we derive the analysis model to investigate the performance of BEBS. Using the analysis model, we can obtain the optimal value of target reliability maximizing the effective capacity of BEBS. Also, we validate the analysis model by comparing the analysis results with the simulation results. Both analysis and simulation results confirm that loosening the target reliability significantly improves the capacity of BEBS; compared to the conventional reliable broadcast, the BEBS improves the capacity by up to about 4 times. Moreover, these results reveal two interesting points; (i) under the typical network configurations, the BEBS is always beneficial in terms of the achievable capacity, regardless of the value of target reliability, (ii) the optimal value of target reliability and the maximum gain of BEBS are almost immune to the number of users, but mostly depend on the distribution of users.

The rest of the paper is organized as follows. In Section II, we describe the system model and service scenario of BEBS and formulate the problem. In Section III, we present the analysis model. Section IV presents analysis results along with simulation results, and the conclusion follows in Section V.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. Service scenario and system model of BEBS

We consider that the BEBS can be served according to the following scenario. The BS serves the BEBS in a single macro-cell using a certain amount of wireless resource that can be either statically reserved or dynamically allocated. Also, the BEBS can be locally served in several micro-cells that are deployed as overlay networks within the coverage of a macro-cell and geographically separate from each other. For example, the BEBS can be used to deliver location-based advertising

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TABLE I
SET OF AVAILABLE MCSs AND CORRESPONDING CELL COVERAGE

<table>
<thead>
<tr>
<th>Index (m)</th>
<th>Modulation (M&lt;sub&gt;m&lt;/sub&gt;-QAM)</th>
<th>Code rate (C&lt;sub&gt;m&lt;/sub&gt;)</th>
<th>Cell coverage (R&lt;sub&gt;m&lt;/sub&gt;)</th>
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</tr>
<tr>
<td>10</td>
<td>64-QAM</td>
<td>5/6</td>
<td>167 m</td>
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</table>

to users in the shopping malls, movie trailers to users near theaters, and live broadcasts to users in sports stadiums.

We consider a general case in which a BS transmits the broadcasting data to N users in the cell, regardless of whether the BEBS is served in the macro-cell or micro-cell. In addition, we consider that the wireless cellular network employs orthogonal frequency division multiple access (OFDMA). Table I lists the set of available MCSs considered in this paper, which mostly agrees with the typical configuration of next generation cellular networks (e.g., Mobile WiMAX or LTE). We define m (1 ≤ m ≤ M) as the index of MCS, where M is the largest index (e.g., M = 10). Note that the cases of m = 1 and m = M correspond to the most robust MCS and the most aggressive MCS, respectively. For each MCS m, information bits are encoded with the convolutional encoder of rate C<sub>m</sub> and the coded bits are interleaved and then mapped to symbols from one of the allowed constellations (M<sub>m</sub>-QAM, where M<sub>m</sub> = 4, 16, 64). We consider that a variable size of downlink burst consisting of several OFDMA symbols and sub-channels is allocated to support the BEBS, and its size depends on the data rate of broadcasting contents and MCS selected by the BS. Defining the data rate of the BEBS and the frame duration as D and T<sub>f</sub>, respectively, B (= DT<sub>f</sub>) bits should be transmitted during a frame for the BEBS. The amount of resources per frame required to support the BEBS with the MCS m, denoted as N<sub>res</sub><sup>m</sup>, can be represented as

\[ N_{\text{res}}^m = \frac{B}{N_{\text{sc}}C_m \log_2 M_m}, \]

where N<sub>sc</sub> is the number of sub-carriers per sub-channel.

B. Problem formulation

The objective of BEBS is to maximize the effective capacity, which is considered to be proportional to the number of users that successfully receive the broadcast data and inversely proportional to the amount of resources used to deliver the data. Unlike the conventional reliable broadcast that mainly uses the robust MCS, the BEBS selects MCS in an aggressive way by intentionally ignoring some users in the poor channel condition (e.g., located at the cell edge). For this purpose, we define target reliability, denoted as η (0 < η ≤ 1), such that the number of users that successfully receive the broadcast data should be at least ηN<sup>1</sup>. Throughout this study, we define that a user receives broadcast data successfully if the average error rate of B-bit burst is less than a certain tolerable error rate ε. This definition is reasonable for real-time multimedia service, which is considered as the major application of BEBS.

In addition, we define N<sub>suc</sub>(η) and N<sub>res</sub>(η), as the average number of successful users and the average amount of resources for the BEBS with a given η, respectively, each of which accounts for the reliability and efficiency. From the economic viewpoint, the problem of maximizing channel capacity under the given reliability constraint can be formulated as

\[
\text{maximize} \quad \frac{N_{\text{suc}}(\eta)}{N_{\text{res}}(\eta)},
\]

subject to \( N_{\text{suc}}(\eta) \geq \eta N \), \( 0 < \eta \leq 1 \).

It is worth noting that the numerator of the objective function in (1) is related to the profit that can be made by the BEBS, while the denominator is related to the cost that should be paid for the BEBS. We can expected that decreasing η decreases both \( N_{\text{suc}}(\eta) \) and \( N_{\text{res}}(\eta) \), i.e., there occurs a trade-off between reliability and efficiency in setting η, and that there exist an optimal value of η that maximizes the given objective function.

III. ANALYSIS MODEL

In this section, we derive the analysis model to investigate how the objective function in (1) is affected by η. First, we derive the cell coverage for each MCS within which the average error rate becomes less than the tolerable error rate. Next, we calculate \( N_{\text{suc}} \) and \( N_{\text{res}} \) to evaluate the effect of η.

A. Derivation of cell coverage

We define the cell coverage R<sub>m</sub> for MCS m as

\[ R_m = \max\{r|\bar{p}_m(r) < \varepsilon\}, \]

where r is the distance between the BS and a user and \( \bar{p}_m \) is the average error rate of B-bit burst with MCS m. The underlying assumption behind (2) is that \( \bar{p}_m \) mostly depends on the large-scale path-loss attenuation, and thus, it increases monotonically with respect to the value of r.

Now, we derive \( \bar{p}_m \) as a function of r. According to the IEEE 802.16m evaluation methodology [7], the path-loss attenuation with distance r kilometers, PL(r), in urban areas at a carrier frequency of 2.5 GHz can be modeled as

\[ PL(r) \ [\text{dB}] = 130.19 + 37.6 \log_{10} r. \]

Let us denote the transmission power of the BS and the background noise power in dBm as P<sub>tx</sub> and P<sub>noise</sub>, respectively. Then, the average value of signal-to-noise ratio (SNR), \( \bar{\gamma} \), can be expressed as

\[ \bar{\gamma} \ [\text{dB}] = P_{tx} - PL(r) - P_{\text{noise}}. \]

In addition, we consider the Rayleigh fading model to account for the small-scale fading caused by non-line-of-sight signals with different delays. The probability density function of the
depends on the distance between BS and user (γ)
\[ \text{(7)} \]
where \( \bar{\gamma} \) is the average SNR determined from the large-scale path-loss attenuation as in (4).

For each MCS and its corresponding modulation scheme, we can represent bit error rate (BER), \( b_m \), in terms of SNR, γ, over the additive white Gaussian noise (AWGN) channel [8]. In the case of \( M_m \)-ary QAM with Gray-coded assignment, \( b_m \) becomes
\[ \text{(6)} \]
where \( s_m \) is the symbol error probability given as
\[ s_m = 1 - \left[ 1 - \left( 1 - \frac{1}{\sqrt{M_m}} \right) Q \left( \sqrt{3} M_m^{-1} \right) \right]^2, \]
where \( Q(\cdot) \) is the complementary Gaussian error function. We assume that the channel status does not change significantly during one frame and the hard-decision Viterbi decoding algorithm is used at the receiver. Then, the error probability for a \( B \)-bit burst with MCS \( m \), denoted as \( p_m^e \), can be approximated as
\[ p_m^e \approx 1 - \left( 1 - p_m^u(b_m) \right)^B. \]
\[ \text{(7)} \]
Here, \( p_m^u(b_m) \) is the union bound of the first-event error probability and can be obtained with the hard-decision decoding [9]. For a given \( B \), we can see that the error probability \( (p_m^e) \) in (7) is a function of BER \( b_m \), which can be represented in terms of SNR (γ). Thus, the average error probability can be calculated as
\[ \bar{p}_m = \int_0^\infty p_m^e(\gamma)p(\gamma)d\gamma. \]
\[ \text{(8)} \]
Note that \( \bar{p}_m \) is determined by \( \bar{\gamma} \) as shown in (5) – (8). Since \( \bar{\gamma} \) depends on the distance between BS and user \( r \) as in (3) and (4), \( \bar{p}_m \) can be obtained from r. Fig. 1 shows \( \bar{p}_m \) versus \( r \) when \( D = 1 \text{ Mb/s} \) and \( T_f = 5 \text{ ms} \). This figure confirms that \( \bar{p}_m \) is an increasing function of \( r \) and can be used to determine the cell coverage \( (R_m) \) once \( \varepsilon \) is given. Table I lists \( R_m \) obtained in this way for \( \varepsilon = 0.05 \).

B. Analysis of reliability and efficiency

In this subsection, we analyze the effect of \( \eta \) on \( \bar{N}^{\text{succ}}(\eta) \) and \( \bar{N}^{\text{res}}(\eta) \) for a given spatial distribution of users. Let us define \( P_m \) as the probability that a user is located with the distance from the BS of \( r \) such that
\[ P_m = \left\{ \begin{array}{ll}
\text{Prob} \left[ R_{m+1} < r \leq R_{m} \right], & \text{if } m = 1, 2, \cdots, M - 1, \\
\text{Prob} \left[ r \leq R_{m} \right], & \text{if } m = M,
\end{array} \right. \]
\[ \text{(9)} \]
which can be determined from the spatial distribution of users. Note that \( R_{m+1} < R_{m} \), i.e., \( R_m \) decreases if the more aggressive MCS is used. We also define \( P_{\text{pin}}^m \) and \( P_{\text{pout}}^m \) as the probabilities that a user is located such that \( r \leq R_{m+1} \) and \( r > R_{m} \), respectively, i.e.,
\[ P_{\text{pin}}^m = \sum_{i=m+1}^{M} P_i, \quad m = 1, 2, \cdots, M - 1, \]
\[ P_{\text{pout}}^m = \sum_{i=1}^{m-1} P_i, \quad m = 2, 3, \cdots, M. \]
\[ \text{(10)} \]
We assume that when MCS \( m \) is used, a user receives the broadcasting data successfully as long as \( r \leq R_m \), otherwise if \( r > R_m \), it fails to do. Thus, \( P_{\text{pout}}^m \) represents the probability that a user fails to receive the data with a given MCS \( m \). Let us denote \( N_f(\eta) \) as the maximum allowable number of users that fail to receive the broadcast data, i.e.,
\[ N_f(\eta) = \lfloor N \cdot (1 - \eta) \rfloor, \]
where \( \lfloor x \rfloor \) is the largest integer that does not exceed \( x \).

Since BS can determine MCS such that the number of unsuccessful users does not exceed \( N_f(\eta) \), the probability that BS selects MCS \( m \), \( P_{\text{sel}}^m(\eta) \), can be obtained as
\[ P_{\text{sel}}^m(\eta) = \left\{ \begin{array}{ll}
\sum_{k=N_f(\eta)+1}^{N} \binom{N}{k} (P_{\text{pin}}^m)^{N-k}(P_m)^k, & \text{for } m = 1, \\
\sum_{k=0}^{N_f(\eta)} P_m^k(k), & \text{for } 2 \leq m \leq M.
\end{array} \right. \]
\[ \text{(11)} \]
Here, \( P_m^k(k) \) denotes the probability that BS selects MCS \( m \) under the condition that the number of unsuccessful users is \( k(0 \leq k \leq N_f(\eta)) \), which can be represented as
\[ P_{m}^k(k) = \left\{ \begin{array}{ll}
\binom{N}{k} (P_{\text{pout}}^m)^{k} \times \sum_{j=N_f(\eta)+1}^{N-k} \binom{N-k}{j} (P_m)^j (P_{\text{pin}}^m)^{N-k-j}, & \text{for } 2 \leq m \leq M - 1, \\
\binom{N}{k} (P_m)^{N-k}(P_{\text{pout}}^m)^k, & \text{for } m = M.
\end{array} \right. \]
\[ \text{(12)} \]
From (9)–(12), \( \bar{N}^{\text{succ}}(\eta) \) and \( \bar{N}^{\text{res}}(\eta) \) can be calculated as
\[ \bar{N}^{\text{succ}}(\eta) = N P_{\text{sel}}^1(\eta) + \sum_{m=2}^{M} \sum_{k=0}^{N_f(\eta)} (N - k) P_{m}^k(k), \]
\[ \text{(13)} \]
\[ \bar{N}^{\text{res}}(\eta) = \sum_{m=1}^{M} \bar{N}^{\text{res}} P_{\text{sel}}^m(\eta). \]
\[ \text{(14)} \]
Finally, the objective function in (1) can be evaluated from (13) and (14).

2This assumption is made for the tractability of analysis, but dropped in the simulations in the next section.
mapping (EESM) [7]. If the burst error rate is less than the burst error rate by means of exponential effective SNR loss and Rayleigh fading models, and then, it is used to get instantaneous value of SNR is determined from the path-simulations as follows. For each user and each frame, the largest MCS index such that

\[ \bar{p}_m^e \leq \bar{\gamma} \]

are sorted in the decreasing order of the SNR and selects the largest MCS. Then, the BS makes the list of users that determine MCS. Then, the BS maintains the mapping table between \( \bar{p}_m^e \) of \( \gamma \)-percentile of user does not exceed \( \varepsilon \) (=0.05). In this way, the reliability constraint of BEBS can be satisfied. Also, \( N^{\text{succ}} \) is determined in the simulations as follows. For each user and each frame, the instantaneous value of SNR is determined from the path-loss and Rayleigh fading models, and then, it is used to get the burst error rate by means of exponential effective SNR mapping (EESM) [7]. If the burst error rate is less than \( \varepsilon \), we consider that the user successfully receives the given frame. One instance of simulation is carried out for \( L (=1000) \) frames. After the simulation is terminated, \( N^{\text{succ}} \) is determined by counting the number of users who successfully received at least \( (1 - \varepsilon) L \) frames. The simulation results are averaged over ten instances of simulation. The simulation results are more realistic than the analysis results from the following points; (i) the analysis calculates the probability distribution of MCS from the probability density function of user’s position, while the simulation determines and adjusts the MCS from the actual SNR of users \(^3\), (ii) the success of frame reception depends solely on user’s position in the analysis, however it is determined from the instantaneous value of SNR in the simulation, i.e., it may change on a frame-by-frame basis in the simulation.

We establish the following performance indices to observe the effect of \( \eta \) on the performance:

- Reliability: We measure the reliability in terms of \( \bar{R}^{\text{succ}}(\eta) = \bar{N}^{\text{succ}}(\eta)/N, \) i.e., the ratio of users that successfully received the broadcasting data among all the users.
- Efficiency: Instead of \( N^{\text{res}}(\eta) \), the efficiency is measured by \( C^{\text{eff}}(\eta) = B/N^{\text{res}}(\eta) \). This is the average number of bits that can be served in a frame with a unit OFDMA resource consisting of one symbol and one sub-channel.
- Capacity: As a measure of capacity, we define a relative gain of BEBS by taking both reliability and efficiency into consideration as:

\[ G(\eta) = \frac{\bar{N}^{\text{succ}}(\eta)/N^{\text{res}}(\eta)}{\bar{N}^{\text{succ}}(1)/N^{\text{res}}(1)}, \]

i.e., the value of the objective function with the given \( \eta \) as defined in (1) divided by that with \( \eta \) of 1 (i.e., fully reliable broadcasting).

As shown in Fig. 2(a), \( \bar{R}^{\text{succ}}(\eta) \) monotonically increases with respect to \( \eta \) and it is always larger than \( \eta \) (denoted as dotted straight line) for the entire range of \( \eta \), regardless of the user distribution, confirming that the BEBS satisfies the reliability constraint. There are no remarkable differences (i) between the analysis results and simulation results and (ii) between the case of uniform distribution and the case of RWPM distribution.

\(^3\)The simulation assumes that the feedback of channel status is ideal, i.e., the delay or error in the channel status feedback is not considered.
Fig. 2(b) shows that $C_{\text{eff}}(\eta)$ monotonically decreases as $\eta$ increases, implying that the efficiency is degraded to achieve the required reliability. Unlike $R_{\text{suc}}^{\text{res}}(\eta)$, the value of $C_{\text{eff}}(\eta)$ in the case of RWPM distribution is notably higher than that in the case of uniform distribution, which stems from the aggressive MCSs of the RWPM distribution.

Next, we focus on the relative gain of BEBS, $G(\eta)$. Note that $G(\eta)$ in (16) can be rewritten as

$$
G(\eta) = \left( \frac{\bar{N}_{\text{suc}}(\eta)}{\bar{N}_{\text{suc}}(1)} \right) \left( \frac{\bar{N}_{\text{res}}(1)}{\bar{N}_{\text{res}}(\eta)} \right).
$$

(17)

For $\eta < 1$, the first term in the right side of (17) is less than one and it evaluates the loss from which BEBS suffers due to the decrease in the number of successful users, while the second term becomes larger than one and it accounts for the achievable gain by BEBS due to the increased efficiency of channel resource. Fig. 3 shows $G(\eta)$ for various values of $\eta$ and reconfirms that the analysis results agree well with the simulation results. As $\eta$ increases to one, $G(\eta)$ also approaches one. It is important to note that $G(\eta)$ is always higher than one for the entire range of $\eta$ ($0.1 < \eta < 1$), i.e., whatever value $\eta$ has, the BEBS outperforms the conventional reliable broadcasting in terms of capacity. Moreover, we can see that $G(\eta)$ in the case of RWPM distribution is always higher than that in the case of uniform distribution. This means that the gain of BEBS is magnified when users gather around the BS. From Fig. 3, we also observe that there exists the optimal value of $\eta$ maximizing $G(\eta)$. Let us denote $G^*$ and $\eta^*$ as the maximum value of $G(\eta)$ and the optimal value of $\eta$, respectively. Table II lists $G^*$ and $\eta^*$ obtained from analysis and simulation for various values of $N$. In the case of uniform distribution, $G^*$ and $\eta^*$ are about 2.3 and 0.3, respectively, while $G^*$ increases up to about 3.8 when $\eta^*$ is about 0.4, in the case of the RWPM distribution. Also, the results in Tab. II confirm that $G^*$ and $\eta^*$ are almost immune to the number of users, but mostly depend on the user distribution. This property is a strong point of BEBS from the viewpoint of practical deployment; once the user distribution is determined by experiments, the optimal target reliability can be set, regardless of the number of users in the cell.

Furthermore, we carried out extensive simulations with different values of $\varepsilon$ (tolerable error rate) and $D$ (data rate of broadcasting), and observed that they do not have any significant effect on all these three performance indices.

### V. Conclusion

We have suggested the service model of BEBS in wireless cellular networks and have formulated its problem as maximizing channel capacity by relaxing the reliability constraint. We have derived the analysis model to investigate the effect of target reliability on the achievable capacity of BEBS. Along with simulation results, the analysis results have verified the potential gain of BEBS. Future work will include elaborating the analysis for deriving upper/lower bounds on the gain of BEBS and the optimal value of target reliability and devising an elaborate control mechanism for realizing the gain of BEBS.

### ACKNOWLEDGMENT

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### TABLE II

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