Optimization Approach for Throughput Analysis of Multi-hop Wireless Networks

Chaegwon Lim
Ministry of Science, ICT, and Future Planning
Republic of Korea

Chong-Ho Choi
Seoul National University
Seoul, Republic of Korea

Hyuk Lim*
Gwangju Institute of Science and Technology
Gwangju, Republic of Korea

Kyung-Joon Park
Daegu Gyeongbuk Institute of Science and Technology
Daegu, Republic of Korea

***Abstract*—A critical performance metric of multi-hop wireless networks is the end-to-end throughput of a multi-hop wireless path. When the packets of a traffic flow originating from a source node are relayed by intermediate nodes (relay nodes), competition between the nodes on the multi-hop routing path incurs transmission delays from the carrier sensing mechanism, as well as packet drops and retransmissions due to hidden terminals. Owing to this complicated interaction between nodes, problems in the throughput performance analysis of multi-hop networks have not been fully resolved. In this paper, we propose an optimization approach to compute the throughput at each individual link in addition to the end-to-end throughput in an IEEE 802.11 DCF-based multi-hop wireless network. To this end, we first analyze a multi-hop chain topology with a single flow by taking into account transmission collisions from hidden terminals, and then formulate the optimization problem to obtain the end-to-end throughput performance of multi-hop networks. Through extensive ns-2 simulations, we then show that the proposed approach accurately predicts the throughput performance obtained by the simulations, within a discrepancy of less than 10–12 % in the worst case scenario under a variety of chain and cross topologies.**

***Index Terms*—Multi-hop wireless networks, end-to-end throughput, IEEE 802.11 WLAN, performance analysis.**

I. Introduction

Multi-hop wireless networks have received considerable attention in recent years as a new, cost effective, and performance-adaptive networking technology, primarily because of their potential in a wide-range of civilian and military applications, as well as their inherent ability to be used without a pre-existing infrastructure. Multi-hop wireless networks consist of a number of either stationary or mobile wireless nodes, which serve as relays to forward traffic from other nodes (along with their own traffic) and maintain network-wide connectivity. In other words, the packets of a flow originating from a source node are relayed by intermediate nodes (relay nodes) and travel toward their destination along a multi-hop wireless path. In this manner, any pair of nodes in a multi-hop wireless network can exchange packets.

However, one-hop wireless networks such wireless local area networks (WLANs) are currently very popular and are widely used on campuses, in offices, and in public locations. To date, the deployment of multi-hop wireless networks has been quite limited owing to an unsatisfactory throughput performance, which is caused by factors such as extremely volatile path properties [1], [2], excessive number of dropped packets [3], [4], and intra- and inter-flow interferences [5].

To improve the overall network throughput in multi-hop wireless networks, extensive studies and analyses of throughput performance have been carried out. These analyses have enabled the development of high-throughput multi-hop networks from a sound theoretical basis, instead of taking a more heuristic approach. For instance, Gupta and Kumar [6] and Jain *et al.* [7] analyzed the theoretical capacity bound of wireless networks using a simplified protocol model. The authors in [8]–[10] derived the throughput performance of one-hop wireless networks in consideration of IEEE 802.11 DCF operation. Ng and Liew [13], [14] then analyzed the maximum sustainable throughput in multi-hop wireless networks based on the IEEE 802.11 DCF, under the assumption that the airtimes are the same for all links. In addition, Gao *et al.* [15] derived the throughput value for each link on the path, and determined the maximum end-to-end throughput as being the value of the lowest throughput on the path. A more detailed summary of related work is given in Section II.

The primary aim of this paper is to determine the following: 1) throughput at each individual link, and 2) end-to-end throughput of multi-hop paths, in an IEEE 802.11 DCF-based multi-hop wireless network. To achieve these tasks, we first analyze a multi-hop chain topology with a single flow, taking into account transmission collisions occurring from hidden terminals. Then, based on this analytical model, we formulate an optimization problem to maximize the throughput of the last link under a set of constraints derived from the interference relationship among competing nodes for k-hop chain topologies. It should be noted that our study is different from the approaches in [13]–[15] in that we do not make the assumption that all nodes have the same airtime in a steady state, and that the maximum end-to-end throughput is determined based on the airtime of the link having the lowest throughput on the path. Through extensive ns-2 simulations, this approach is then evaluated in diverse networking scenarios with different payload sizes, signal attenuation factors, and network topologies. The simulation results show that the discrepancy between the end-to-end throughput obtained by the analysis and the ns-2 simulations is within 12 % for a chain topology network with one traffic flow and within 10 % in a six-hop cross topology network with two traffic flows.

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The rest of this paper is organized as follows. In Section II, we present a summary of existing related work, and highlight the limitations of current state-of-the-art approaches. In Section III, we present an analytical model for collisions owing to the hidden terminal problem in a multi-hop wireless network, and In Section IV we formulate an optimization problem to determine the maximum end-to-end throughput in a multi-hop wireless path. In Section V, we show our simulation studies, which verify the estimation accuracy of the throughput in both chain and cross topologies. Finally, Section VI provides some concluding remarks, along with a description of several potential areas of future research.

II. Related Work

Gupta and Kumar [6] analyzed the capacity of wireless networks, and showed that the per-node throughput capacity is bounded by $\Theta(\frac{1}{\sqrt{n}})$ when n nodes are optimally placed, and by $\Theta(\frac{1}{\sqrt{n \log n}})$ when the nodes are placed randomly. Jain *et al.* [7] also studied the maximum throughput performance in multi-hop wireless networks using a conflict graph when the node placement and traffic workload are given. However, these studies only focused on identifying the *theoretical* optimal capacity of networks by assuming optimal network control and ignoring the underlying MAC properties.

The performance of the IEEE 802.11 DCF in one-hop networks has been previously studied using various approaches, such as the discrete Markov chain model by Bianchi [8], and the p-persistent model by Calì *et al.* [9]. In addition, a more generalized model based on a fixed point analysis was proposed by Kumar *et al.* [10]. Garetto *et al.* [11] identified the interference interactions between two adjacent flows in wireless networks. Based on the observed interactions, they performed a formal analysis to obtain the throughput performance. In [12], Razak *et al.* generalized the analysis performed in [11], and then presented the interference interactions between two flows in a number of different ways. However, because these models focus on the effects of contention collisions and do not take into account the hidden node problem, common in multi-hop networks, they cannot be applied directly.

Recently, several analytical studies on end-to-end throughput in multi-hop networks based on the IEEE 802.11 DCF have been presented. For example, Ng and Liew [13], [14] analyzed the sustainable maximum throughput in chain topology networks. They defined the airtime (S_i) of a node N_i as the sum of the transmission time for the frames from N_i, the transmission time for ACKs from N_{i+1}, the durations of DIFS, and the durations of SIFS; S_i also includes the time used for retransmissions, but excludes the backoff duration. The authors also defined the normalized airtime x_i as $|S_i|/Time$, where $Time$ is the sampling time interval, and x_i is assumed to be the same for each node in the network. Then, they found a value of x^* that can provide maximum network throughput, and subsequently calculated the sustainable throughput $\mathcal{T}(x^*)$. Gao *et al.* [16] then improved upon this model in [13] and derived the optimal hop distance in chain topology networks by applying a fixed point analysis [10]. It should be noted, however, that the above studies do not take into account the different levels of interference at each node, but rather assumed that x_i is the same for all N_i. Unfortunately, this assumption does not hold true in real multi-hop wireless networks. As a result, their approach does not provide an accurate estimate of the throughput performance for short chain networks, where x_i can dramatically decrease with respect to the hop-count on a multi-hop path.

In an attempt to further improve this model, Gao *et al.* [15] extended the model in [16] by relaxing the assumption that x_i is the same for all nodes. However, the throughput of each link was still computed using a flow constraint, assuming that links on the same path should have the same throughput; the authors determined the end-to-end throughput as the lowest throughput on the path. Based on the above considerations, the solutions presented in [15] have a tendency to provide a lower throughput for nodes that are closer to the source node. For instance, in a four-hop chain topology, the first node has the lowest throughput at 1.72 Mb/s, whereas the second, third, and fourth nodes have throughputs of 1.78, 1.78, and 2.24 Mb/s, respectively. However, for a single traffic flow, nodes cannot have a higher throughput than the preceding node on a multi-hop path. Razak *et al.* [17], [18] analyzed the performance of a single chain network and characterized the different types of interference interactions. In [19], Garetto *et al.* presented Information Asymmetry (IA) and Flow-in-the-Middle (FIM) to predict the per-flow throughput in arbitrary networks by estimating the portion of busy time by each node and the probability of a successful transmission. Carvalho *et al.* [20] provided an analytical model for multi-hop networks with multiple senders and receivers. In this model, the authors analyzed the system performance by considering various physical layer parameters. Recently, Zhao *et al.* [21] considered both neighboring interference and hidden node problems to compute the end-to-end throughput of a multi-hop network. The authors also considered a multi-rate scenario for a throughput analysis of this network type.

III. Multi-hop Network model

As the basis of our model, we consider a multi-hop wireless network composed of IEEE 802.11 DCF nodes; IEEE 802.11 DCF is based on carrier sense multiple access with collision avoidance (CSMA/CA) [22]. Fig. 1 shows an example of a chain topology with seven nodes (k = 6), where N_0 and N_k are the source and destination nodes, respectively. In this model, the traffic flow is generated from N_0, and the intermediate nodes between N_0 and N_k then buffer and forward the traffic toward N_k. The distance between two adjacent nodes is set to be within the transmission range (d_T), and the carrier sensing range (d_S) is determined using the value of the carrier sensing threshold. In the analysis, d_S is set to $2.2 \times d_T$; the interference range (d_I) depends on the values of the capture threshold α and path loss attenuation factor β, which are between two and four [23], (i.e., $d_I = \sqrt[\beta]{\alpha} \times d_T$).

In a multi-hop chain topology, the collisions due to simultaneous transmissions depend on the number of nodes in the

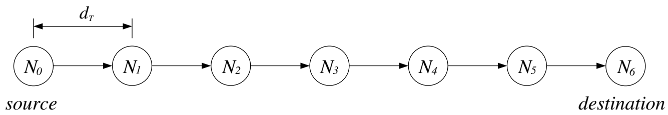

Fig. 1. Multi-hop wireless network with six-hop chain topology.

carrier sensing range (d_S). In Fig. 1, the maximum number of competing nodes is five, (e.g., N_3 competes with N_1, N_2, N_4, and N_5), and the channel attempt probability with which a node starts a transmission at a certain slot is 1/16; the attempt probability is given by $\frac{2}{CW+1}$ when $CW = CW_{min}$, where CW and CW_{min} are the contention window value and the minimum value of CW, as stated in [8]–[10]. Based on these factors, the corresponding collision probability is computed to be approximately 0.034. These results indicate that in a multi-hop chain topology, collisions owing to simultaneous transmissions are negligible, and transmission failures are more likely due to hidden nodes.

First, we consider collisions from hidden nodes, as shown in Fig. 1. For α = 10 dB and $\beta = 3.3$, $d_I \approx 2 \times d_T$. In this case, the transmissions from N_0 may be corrupted by transmissions from N_3, as N_3 becomes a hidden node to N_0. Fig. 2 shows timing diagrams for the transmissions of N_0 and N_3 under the IEEE 802.11 DCF without an RTS/CTS exchange. In Cases 1 and 2, while N_3 is transmitting, N_1 simultaneously receives frames from N_0; thus, its transmission will be unsuccessful as N_3 is within the interference range of N_1. However, in Case 3, the transmission of N_0 will be successful as the transmission from N_4 does not interfere with the reception of N_1. Thus, the ratio of transmission failure for N_0 due to interference from N_3 can be given by

$$u = \frac{T_{DIFS} + T_{E[BC]} + T_{DATA}(l)}{T_{FRAME}(l)}. \quad (1)$$

Here, $T_{FRAME}(l)$ is the amount of time spent to complete a data transmission, and is given by

$$T_{FRAME}(l) = T_{DIFS} + T_{E[BC]} + T_{DATA}(l) + T_{SIFS} + T_{ACK},$$

where l is the size of a DATA frame, T_{DIFS} and T_{SIFS} are the interframe spaces defined in the IEEE 802.11 standard, $T_{E[BC]}$ is the backoff time, and T_{DATA} and T_{ACK} are the transmission times for the DATA and ACK frames, respectively. It should be noted here that our model does not include the binary exponential backoff (BEB) mechanism of IEEE 802.11 DCF, and that $E[BC]$ is given by $CW_{min}/2$. (we will further discuss the effect of this simplification in Section V). In Fig. 2, the ratio of transmission failure is derived under the assumption that the $N_3 - N_4$ link is fully utilized. Therefore, if the airtime (denoted by x_3) is lower than 1, the transmission failure rate at N_0 is given by $u \cdot x_3$ owing to the hidden terminal.

However, for the path loss factor $\beta = 4.0$, d_I becomes $1.78 \times d_T$. This implies that N_3 will no longer interfere with N_1 because it is beyond the interference range of N_1, as shown in Fig. 1. The hidden node problem occurs when the receiver

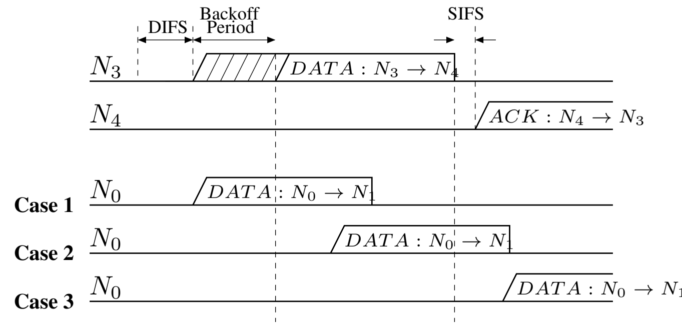

Fig. 2. Timing diagrams for two concurrent transmissions under IEEE 802.11 DCF with an RTS/CTS exchange. The transmission from N_3 interferes with the data reception at N_1 in Cases 1 and 2.

cannot decode the signals from the sender, as is the case when the receiver has already tuned to another transmission [15]. Let us now consider Case 2 in Fig. 2. When N_3 begins to transmit, N_1 senses the signals from N_3 and tunes to this transmission, although it cannot correctly decode the signal. At this point, even though N_0 starts to transmit to N_1, N_1 cannot capture the signals from N_0, as it has already tuned to the transmission from N_3. Consequently, no ACK frame will be sent to N_0, which in turn is interpreted as a collision by N_0. Note that Cases 1 and 3 in Fig. 2 do not cause a transmission failure in N_1. In this case, u becomes

$$u = \frac{T_{DATA}(l)}{T_{FRAME}(l)}. \quad (2)$$

The ratio of transmission failure is derived for a simple chain topology where only one hidden terminal exists. When there are multiple hidden terminals, the failure ratio increases by $\sum_{\mathcal{H}} u \cdot x_{i,h}$, where $\mathcal{H}$ is the set of hidden nodes, and $x_{i,h}$ is the airtime of these hidden nodes for N_i. Note that, for simplicity, we ignore the interactions among the hidden nodes themselves.

IV. Optimization based Throughput Analysis

The normalized airtime x_i is defined as a portion of the transmission airtime of N_i over unit time t in a steady state as $t \rightarrow \infty$ [13]–[16]. Note that x_i includes the transmission time and MAC overhead, including the back-off time for transmission contention. By definition, the throughput of N_i is given by

$$\mathcal{T}_i(l) = x_i \; (1 - \rho_i) \; \frac{l}{T_{FRAME}(l)}, \quad (3)$$

where ρ_i is the collision probability. The collision probability ρ_i is calculated as $\frac{u \cdot \sum_{\mathcal{H}} x_{i,h}}{1 - \sum_{\mathcal{C}} x_{i,c}}$, where $\mathcal{C}$ denotes the set of common nodes for link i in [14], [15]. Note that any node $n \in \mathcal{C}$ cannot transmit while N_i is transmitting as such nodes are within the same carrier sensing range. In this analysis, it is assumed that a collision only occurs due to transmissions from hidden nodes. The end-to-end throughput achievable on a k-hop wireless path is

$$\mathcal{T}(l) = x_{k-1} \; \frac{l}{T_{FRAME}(l)} \quad (4)$$

under the assumption that ρ_{k-1} is negligible at the last hop. This assumption is based on the fact that collisions on a multi-hop wireless path are mainly due to hidden terminals, and that the last hop has no hidden terminals. To compute the maximum end-to-end throughput, we propose an **optimization approach for maximizing the airtime of the last link** x_{k-1} with a set of constraints derived from the interference relationship among the competing nodes.

1) One-hop chain topology: In a one-hop chain topology, N_0 is the sender and N_1 is the receiver. The optimization problem for this topology can be formulated as

$$\begin{aligned} \text{maximize} \quad & x_0 \\ \text{subject to} \quad & x_0 \leq 1. \end{aligned} \tag{5}$$

The constraint, $x_0 \leq 1$, implies that the maximum of x_0 is 1 as there are no other competing nodes. Thus, the solution to (5) is simply $x_0 = 1$, implying that the maximum throughput is achieved when N_0 is continuously transmitting.

2) Two-hop chain topology: In a two-hop chain topology, N_0 is a sender, N_2 is a receiver, and they both compete with each other. Note that x_1 cannot exceed x_0 because N_1 cannot send out more than the amount of data received from N_0. Here, the objective function is to maximize x_1 as the end-to-end throughput is the receiving rate at the last node, N_2, i.e., the throughput of N_1.

$$\begin{aligned} \text{maximize} \quad & x_1 \\ \text{subject to} \quad & x_0 + x_1 \leq 1, \\ & x_1 \leq x_0. \end{aligned} \tag{6}$$

The solution to (6) is $x_0 = x_1 = \frac{1}{2}$, which implies that the maximum end-to-end throughput is obtained when N_0 and N_1 have equal access to the channel.

3) Three-hop chain topology: The modeling of a three-hop chain topology is similar to that of a two-hop chain topology. Three nodes, N_0, N_1, and N_2, compete for the channel, and there are no hidden nodes. In this case, the objective function is to maximize x_2 as follows.

$$\begin{aligned} \text{maximize} \quad & x_2 \\ \text{subject to} \quad & x_0 + x_1 + x_2 \leq 1 \\ & x_1 \leq x_0 \\ & x_2 \leq x_1. \end{aligned} \tag{7}$$

The solution to this problem is thus $x_0 = x_1 = x_2 = \frac{1}{3}$.

4) Four-hop chain topology: In a four-hop chain topology, the hidden node problem needs to be considered as N_3 is hidden from N_0. Here, the outgoing traffic from N_1 cannot exceed the amount of incoming traffic that has been successfully delivered from N_0. This constraint is expressed as

$$x_1 \leq x_0 \left(1 - \frac{u \times x_3}{1 - x_1 - x_2}\right). \tag{8}$$

Because N_3 is allowed to access the channel only when N_1 and N_2 are idle, the airtime during which N_0 and N_3 may collide is $1 - x_1 - x_2$ [14], [15].

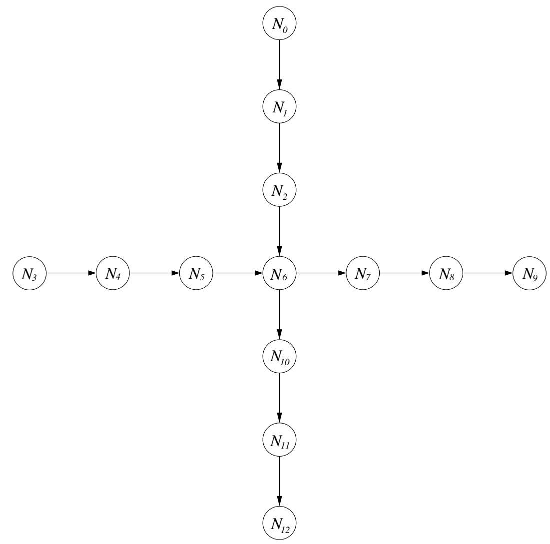

Fig. 3. A multi-hop wireless network with a six-hop cross topology.

Similarly, the optimization for a four-hop network is formulated as

$$\begin{aligned} \text{maximize} \quad & x_3 \\ \text{subject to} \quad & x_0 + x_1 + x_2 \leq 1 \\ & x_1 \leq x_0 \left(1 - \frac{u \times x_3}{1 - x_1 - x_2}\right) \\ & x_2 \leq x_1 \\ & x_3 \leq x_2. \end{aligned} \tag{9}$$

5) k-hop chain topology ($k \geq 4$): In a k-hop chain topology, we further generalize the constraint for hidden nodes as

$$x_{i+1} \left(1 - \frac{u \times x_{i+4}}{1 - x_{i+2} - x_{i+3}}\right) \leq x_i \left(1 - \frac{u \times x_{i+3}}{1 - x_{i+1} - x_{i+2}}\right),$$

for $i = 0, \cdots, (k-5)$. This constraint implies that the amount of outgoing traffic sent by N_{i+1} cannot exceed the amount of incoming traffic from N_i. Thus, because all links other than the last three links suffer from the hidden node problem, the formulation used to find the maximum end-to-end throughput becomes

$$\begin{aligned} \text{maximize} \quad & x_{k-1} \\ \text{subject to} \quad & x_0 + x_1 + x_2 \leq 1 \\ & x_{i+1} \left(1 - \frac{u \times x_{i+4}}{1 - x_{i+2} - x_{i+3}}\right) \\ & \quad \leq x_i \left(1 - \frac{u \times x_{i+3}}{1 - x_{i+1} - x_{i+2}}\right) \\ & \quad \text{for} \quad i = 0, \cdots, (k-5) \\ & x_{k-3} \leq x_{k-4} \left(1 - \frac{u \times x_{k-1}}{1 - x_{k-3} - x_{k-2}}\right) \\ & x_{k-2} \leq x_{k-3} \\ & x_{k-1} \leq x_{k-2}. \end{aligned} \tag{10}$$

TABLE I
HIDDEN AND COMMON NODES FOR THE SIX-HOP CROSS TOPOLOGY IN FIG. 3.

Link	Hidden nodes	Common nodes
$N_3 \to N_4$	N_6	N_4, N_5
$N_4 \to N_5$	N_7	N_5, N_6
	N_2	N_5, N_6
	N_{10}	N_5, N_6
$N_5 \to N_6$	N_8	N_6, N_7
	N_1	N_2, N_6
	N_{11}	N_6, N_{10}
$N_0 \to N_1$	N_6	N_1, N_2
$N_1 \to N_2$	N_{10}	N_2, N_6
	N_7	N_2, N_6
	N_5	N_2, N_6
$N_2 \to N_6$	N_{11}	N_6, N_{10}
	N_8	N_7, N_6
	N_4	N_6, N_5

6) Cross topology: At this point, in multi-hop chain topologies, there has only been one flow in the network, and each N_i is interfered with by only one hidden node. However, in cross topologies, several flows may coexist, and some nodes may have multiple hidden nodes. In this topology, the normalized airtime of node N_r, in which frames can be successfully transmitted, can be calculated as

$$x_r \left(1 - u \times \sum_i \frac{x_{i,r}}{1 - \sum_c x_c}\right), \qquad (11)$$

where $x_{i,r}$ is the airtimes of the nodes $N_{i,r}$, and x_c is the airtime of the node N_c. Note that $N_{i,r}$ are the nodes hidden nodes from N_r, and N_c is the common node within the carrier sense ranges of both N_r and $N_{i,r}$.

Fig. 3 shows an example of six-hop cross topology with two flows; the hidden nodes and common nodes for each link are listed in Table I. For this topology, we impose another constraint in that two flows equally share the channel bandwidth, i.e., $x_8 = x_{11}$. Then, using (11) and the information in Table I, the optimization can be formulated, and here is omitted due to the page limit.

V. VERIFICATION

In this section, the derived analysis for the end-to-end throughput in multi-hop wireless networks is verified through extensive simulations. In this analysis, the ns-2 simulator (version 2.30) is used, and the presented results are the averaged values from ten runs with a simulation time of 100 seconds. Also note that the sender nodes transmit UDP packets with 1000 bytes payload, and that an RTS/CTS exchange is not enabled. The IEEE 802.11b parameters used in the simulation are listed in Table II, and the path loss exponent attenuation factor β is set to 3.3 by default. Finally, the optimization problems are solved using `cvx`, a Matlab-based optimization tool [24].

A. Normalized airtime

First, we need to solve the optimization problem of (10) for a k-hop chain topology, where the number of hops k varies from 1 to 8. Table III shows the normalized airtime of each link for k-hop networks, and it can be seen that the solutions satisfy the original constraints in (10).

TABLE II
IEEE 802.11B PARAMETERS USED IN THE SIMULATIONS AND ANALYSES.

Parameter	Value
T_{DIFS}	50 μsec
T_{SIFS}	10 μsec
T_{Slot}	20 μsec
PHY overhead	192 μsec
Data frame MAC header	28 bytes
Ack frame MAC header	14 bytes
UDP/IP header	20 bytes
Attenuation factor β	3.3
Data transmission rate	11 Mb/s
Ack transmission rate	2 Mb/s
PHY header transmit rate	1 Mb/s
CW_{min}	31
CW_{max}	1023
Buffer size of nodes	50 packets

TABLE III
NORMALIZED AIRTIME OBTAINED USING THE ANALYSIS FOR k-HOP CHAIN TOPOLOGY (PAYLOAD SIZE IS 1000 BYTES).

k	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	1							
2	0.5	0.5						
3	0.33	0.33	0.33					
4	0.47	0.26	0.26	0.26				
5	0.41	0.35	0.22	0.22	0.22			
6	0.38	0.32	0.29	0.20	0.20	0.20		
7	0.41	0.30	0.28	0.26	0.19	0.19	0.19	
8	0.40	0.33	0.27	0.25	0.24	0.18	0.18	0.18

Table IV shows the normalized airtime ($\hat{x}_i$) obtained using ns-2 simulations, as calculated by dividing the total airtime of N_i with the simulation time of 100 seconds. We can see that the analysis provides quite accurate estimates of the normalized airtime, especially for the last three links of the network shown in Tables III and IV. However, the discrepancy between x_i and $\hat{x}_i$ increases for nodes closer to the source.

B. Maximum end-to-end throughput

To verify the correctness of the proposed analysis, the maximum throughput of the linear topologies was investigated for several payload sizes. Table V and Fig. 4 illustrate

TABLE IV
NORMALIZED AIRTIME OBTAINED USING NS-2 SIMULATIONS FOR k-HOP CHAIN TOPOLOGY (PAYLOAD SIZE IS 1000 BYTES).

k	$\hat{x}_0$	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$	$\hat{x}_5$	$\hat{x}_6$	$\hat{x}_7$
1	1.00							
2	0.51	0.51						
3	0.37	0.39	0.38					
4	0.53	0.26	0.25	0.24				
5	0.48	0.40	0.23	0.22	0.21			
6	0.45	0.38	0.31	0.21	0.21	0.20		
7	0.49	0.35	0.29	0.25	0.20	0.19	0.19	
8	0.47	0.38	0.28	0.25	0.23	0.19	0.19	0.18

TABLE V
MAXIMUM END-TO-END THROUGHPUT OBTAINED USING THE ANALYSIS AND NS-2 (KB/S, PAYLOAD SIZE IS 1000 BYTES).

k	Analysis	Simulation	Discrepancy (%)
1	5088.47	5088.62	0.00
2	2676.15	2485.40	7.13
3	1801.39	1789.50	0.66
4	1392.33	1226.72	11.89
5	1213.60	1090.05	10.18
6	1102.68	1050.00	4.78
7	1029.47	991.97	3.64
8	980.41	970.08	1.05
12	880.00	909.92	3.40
16	839.37	891.06	6.15

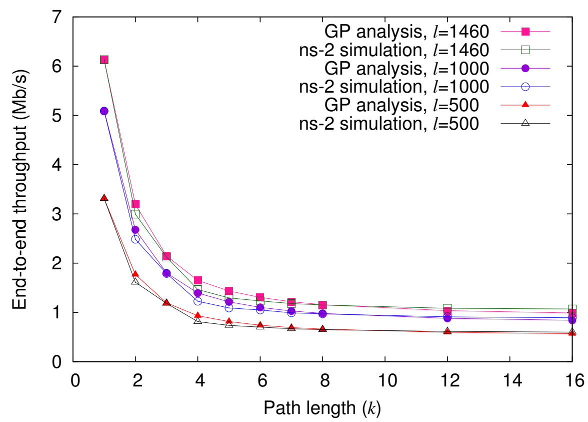

Fig. 4. Maximum end-to-end throughput for payload sizes of 500, 1000, and 1460 bytes ($\beta = 3.3$).

a comparison of the calculated maximum throughput from the analysis with that measured from the ns-2 simulations. Because the end-to-end throughput in a multi-hop wireless path depends on the offered load during the simulation, we can determine the maximum end-to-end throughput by varying the offered load from 10 Kb/s to 6.5 Mb/s at a resolution of 10 Kb/s. In the simulation results, the summation of each airtime within the same carrier sensing range exceeds 1. This is due to the assumption that every node spends the same amount of backoff time, which is computed to be CW/2 without considering the neighboring nodes for simplicity. This problem would be resolved using a more accurate modeling of the backoff mechanism as previously mentioned.

Fig. 4 shows specifically that the maximum end-to-end throughput obtained from the analysis matches quite well that of the ns-2 simulations within a wide range of path lengths k with different payload sizes. In Table V, the largest discrepancy between the maximum end-to-end throughputs of the analysis and those from the simulations is within 11.89 % in a four-hop network, with a payload size of 1000 bytes. From Table V, we can see that in most cases, the throughputs measured through the ns-2 simulations are lower than those from the analysis, which is mainly due to the EIFS of the IEEE 802.11 DCF, which was not considered in the modeling. According to the IEEE 802.11 DCF specifications, if a node fails to decode a frame after its reception, the node assumes that the frame was a data frame and expects an ACK frame to follow. The node then defers its transmission for T_{EIFS}, resulting in a lower throughput performance in the simulation than in the analysis.

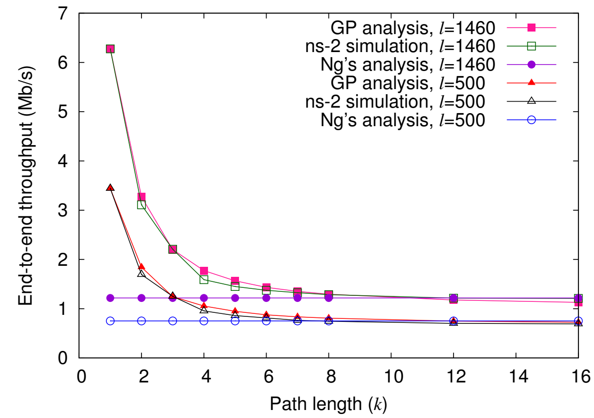

Fig. 5. Comparison of maximum end-to-end throughput for the analysis, ns-2 simulation, and Ng's analysis in [14] ($\beta = 4$).

C. Effect of attenuation factors

We investigated the effects of channel fading, particularly, the effect of path loss attenuation factor β. Attenuation factor β varies between 3.3 and 4. For a larger value of β, the transmission and interference ranges are smaller such that the numbers of competing and hidden nodes are also decreased in chain and cross multi-hop networks. In this case, the ratio of transmission failures u becomes smaller in (2) as the interference range is smaller for $\beta = 4$. Fig. 5 compares the maximum end-to-end throughputs calculated by the analysis and measured through the simulations, and shows that the analysis provides accurate estimates of the end-to-end throughputs for $\beta = 4.0$. We then compared the proposed analysis with an analysis by Ng and Liew in [14], in which the authors derived the end-to-end throughput in multi-hop wireless networks under the assumption that the airtimes of all nodes on the path are the same. Fig. 5 shows that their results are well matched with the measured throughput from the simulations when the number of hops is greater than seven. Conversely, the results obtained from the analysis are well matched with the simulation results for all cases.

D. Cross topology

We performed an ns-2 simulation for the cross topology, the results of which are shown in Fig. 3. We then compared the simulation results to those obtained from the analysis. Table VI shows the end-to-end throughput obtained from the analysis and by the simulations. In the worst case, the discrepancy between the analysis and simulations is less than 9.5 %, regardless of β or the payload size in the cross topology. We then confirmed that in all cases, the optimization solutions with less strict constraints satisfied all original constraints.

TABLE VI
MAXIMUM END-TO-END THROUGHPUT PER FLOW FOR THE CROSS TOPOLOGY IN FIG. 3 (KB/S).

Attenuation factor	Payload (byte)	500	1000	1460
$\beta = 3.3$	Simulation	320.0	462.8	536.5
	Analysis	340.8	501.4	588.8
	Discrepancy	6.1 %	7.7 %	8.8 %
$\beta = 4.0$	Simulation	405.8	552.6	646.3
	Analysis	447.7	607.3	684.6
	Discrepancy	9.4 %	9.0 %	5.6 %

VI. CONCLUSIONS

In this paper, we analyzed the end-to-end throughput of an IEEE 802.11 DCF-based multi-hop network. We formulated an optimization problem to compute the throughput of each node on a multi-hop path, determined using the throughput value obtained at the last hop of the wireless path. The optimization problem was then solved using geometric programming, which could provide a globally optimal solution for the airtime of all links on the path. This method was first applied to a multi-hop network with a chain topology, and, in the worst case, the discrepancy between the end-to-end throughputs obtained from the analysis and the ns-2 simulation was as little as 12 %. In a cross topology, the discrepancy was less than 10 %, which confirms the potential for applying this approach to various wireless networks with a more general topology. In addition, the analysis was successfully applied to estimate the number of transmission attempts required for a successful transmission at the intermediate nodes.

An immediate research agenda is to apply the analysis to more general scenarios with various topologies, including multiple flows and varied transmission rates, as only simple chain and cross topologies were taken into consideration in this work. Once a set of competing and hidden nodes for each node is identified, the analysis is expected to make the optimization problem solvable, allowing us to numerically obtain a high throughput performance in a variety of multi-hop wireless networks. Next, we intend to develop a more detailed mathematical model for IEEE 802.11 DCF in terms of the airtime of links on a multi-hop path. The new model is expected to overcome the limitations of this paper, including detailed behaviors such as the binary exponential backoff (BEB) mechanism and delay owing to EIFS in IEEE 802.11 DCF.

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REFERENCES

[1] S. Xu and T. Saadawi, "Does the IEEE 802.11 MAC Protocol Work Well in Multihop Wireless Ad Hoc Networks?," *IEEE Communications Magazine*, vol. 39, pp. 130–137, June 2001.

[2] K. Sanzgiri, I. D. Chakeres, and E. M. Belding-Royer, "Determining Intra-flow Contention along Multihop Paths in Wireless Networks," in *Proceedings of BROADNETS*, October 2004.

[3] Z. Fu, H. Luo, P. Zerfos, S. Lu, L. Zhang, and M. Gerla, "The Impact of Multihop Wireless Channel on TCP Performance," *IEEE Transactions on Mobile Computing*, vol. 4, pp. 209–221, March/April 2005.

[4] C. Lim, H. Luo, and C. Choi, "RAIN: A Reliable Wireless Network Architecture," in *Proceedings of IEEE ICNP*, November 2006.

[5] H. Lim, C. Lim, and J. C. Hou, "A coordinate-based approach for exploiting temporal-spatial diversity in wireless mesh networks," in *Proceedings of ACM MobiCom*, pp. 14–25, 2006.

[6] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. IT-46, pp. 388–404, March 2000.

[7] K. Jain, J. Padhye, V. Padmanabhan, and L. Qiu, "Impact of Interference on Multi-hop Wireless Network Performance," in *Proceedings of ACM MobiCom*, September 2003.

[8] G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Fucnition," *IEEE Journal on Selected Areas in Communications*, vol. 18, pp. 535 – 547, March 2000.

[9] F. Calì, M. Conti, and E. Gregori, "Dynamic Tuning of the IEEE 802.11 Protocol to Achieve Theoretical Throughput Limit," *IEEE/ACM Transactions on Networking*, vol. 8, pp. 785 – 799, December 2000.

[10] A. Kumar, E. Altman, D. Miorandi, and M. Goyal, "New Insights from a Fixed Point Analysis of Single Cell IEEE 802.11 WLANs," in *Proceedings of IEEE INFOCOM*, March 2005.

[11] M. Garetto, J. Shi, and E. Knightly, "Modeling Media Access in Embedded Two-flow Topologies of Multi-hop Wireless Networks," in *Proceedings of ACM MobiCom*, pp. 200–214, 2005.

[12] S. Razak, V. Kolar and N. B. Abu-Ghazaleh, "Modeling and Analysis of Two-flow Interactions in Wireless Networks," *Ad Hoc Networks, Elsevier*, Volume 8, pp. 564–581, August 2010.

[13] P. C. Ng and S. C. Liew, "Offered Load Control in IEEE802.11 Multi-hop Ad-hoc Networks," in *Proceedings of IEEE MASS*, March 2004.

[14] P. C. Ng and S. C. Liew, "Throughput Analysis of IEEE 802.11 Multi-hop Ad Hoc Networks," *IEEE/ACM Transactions on Networking*, vol. 15, pp. 309–322, April 2007.

[15] Y. Gao, D.-M. Chiu, and J. C. Lui, "Determining the End-to-end Throughput Capacity in Multi-hop Networks: Methodology and Applications," in *Proceedings of ACM SIGMETRICS*, June 2006.

[16] Y. Gao, D.-M. Chiu, and J. C. Lui, "The Fundamental Role of Hop Distance in IEEE 802.11 Multi-hop Ad hoc Networks," in *Proceedings of IEEE ICNP*, November 2005.

[17] S. Razak, V. Kolar, N. B. Abu-Ghazaleh and K. Harras, "How do Wireless Chains Behave? The Impact of MAC Interactions," in *ACM-IEEE International Conference on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM)*, 2009.

[18] S. Razak and N. B. Abu-Ghazaleh, "Self-interference in Multi-hop Wireless Chains," in *Proceedings of ADHOC-NOW*, 2008.

[19] M. Garetto, T. Salonidis, and E. Knightly, "Modeling Per-flow Throughput and Capturing Starvation in CSMA Multi-hop Wireless Networks," *IEEE/ACM Transactions on Networking*, vol. 16, pp. 864–877, August 2008.

[20] M. M. Carvalho and J. J. Garcia-Luna, "A Scalable Model for Channel Access Protocols in Multihop Ad Hoc Networks," in *Proceedings of ACM MobiCom*, pp. 330–344, 2004.

[21] H. Zhao, S. Wang, Y. Xi, and J. Wei, "Modeling Intra-flow Contention Problem in IEEE 802.11 Wireless Multi-hop Networks," *IEEE Communications Letters*, vol. 14, pp. 18-20, January 2010.

[22] IEEE, "Wireless LAN Media Access Control (MAC) and Physical Layer (PHY) Specification," *IEEE Standard 802.11*, 1999.

[23] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Prentice Hall, 1996.

[24] M. Grant, S. Boyd, and Y. Ye, "cvx Users' Guide." http://www.stanford.edu/ boyd/cvx/cvx_usrguide.pdf, February 2007.