QoS Satisfaction Games for Spectrum Sharing

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***Abstract*—Today's wireless networks are facing tremendous growth and many applications have more demanding quality of service (QoS) requirements than ever before. However, there is only a finite amount of wireless resources (such as spectrum) that can be used to satisfy these demanding requirements. We present a general QoS satisfaction game framework for modeling the issue of distributed spectrum sharing to meet QoS requirements. Our study is motivated by the observation that finding globally optimal spectrum sharing solutions with QoS guarantees is NP hard. We show that the QoS satisfaction game has the finite improvement property, and the users can self-organize into a pure Nash equilibrium in polynomial time. By bounding the price of anarchy, we demonstrate that the worst case pure Nash equilibrium can be close to the global optimal solution when users' QoS demands are not too diverse.**

I. Introduction

The number of wireless devices such as smart-phones continues to increase rapidly, while the amount of spectrum available for these devices remains limited. Moreover, many new wireless applications such as the high definition video streaming and the online interactive gaming are emerging, making the quality of service (QoS) of wireless users higher and more diverse. Thus there is an urgent need to study the issue of how to efficiently allocate the limited spectrum to satisfy the QoS demands of as many users as possible.

There are two different approaches towards handling this issue. The first approach is a *centralized* one, where a network operator optimizes the spectrum allocation to meet the users' QoS requirements. This approach puts most of the implementation complexity on the operator's side, and wireless devices do not need to be very sophisticated. However, as networks grow larger and more heterogeneous, this approach may not be suitable for the following two reasons. First, the QoS demands of wireless users are highly heterogeneous, which implies that the operator needs to gather massive amounts of information from users in order to perform the centralized optimization. Second, finding the system-wide optimal QoS satisfaction solution itself is computationally challenging – in fact we show that it is NP hard. It is hence difficult for the operator to compute the optimal solution to meet users' real-time QoS demands. The alternative approach is a *decentralized* one, where each wireless user makes the spectrum access decision locally to meet its own QoS requirement, while taking the network dynamics and other users' actions into consideration. This is feasible since new technologies like cognitive radio [1]–[3] give users the ability to scan and switch channels easily. The decentralized approach enables more flexible spectrum sharing, scales well with the network size, and is particularly suitable when users belong to multiple network operators.

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We investigate the decentralized approach using the framework of congestion game theory. Rosenthal proposed the original congestion game [4] to model how selfish players share heterogenous resources. A player's utility from using a resource depends on the congestion level of that resource, which is the number of players sharing that resource. Congestion games can model spectrum sharing when the players represent wireless users and resources represent channels. However, in many wireless channels users are often highly heterogenous. The congestion games with player-specific utility functions considered in [5] are more appropriate for this general scenario. Authors in [6]–[8] have adopted such game models for studying spectrum sharing problems. In [9]–[13], we considered how *graphical congestion games* can be used to model spectrum sharing. Here the players are represented as vertices in a graph, and each player only interacts with his neighbors. The graph models the interference relationships between the wireless users.

A common assumption in previous congestion game based spectrum sharing literature is that a user's utility strictly increases with its received data rate (and hence strictly decreases with the congestion level). This is true, for example, when users are running elastic applications such as file downloading. However, there are many other types of applications with more specific QoS requirements, such as VoIP and video streaming. These inelastic applications can not work properly when their QoS requirements (such as target data rates) are violated, but do not obtain additional benefits when given more resources than needed. This kind of traffic is becoming increasingly popular over the wireless networks (e.g., mobile video traffic exceeded 50% percent of all wireless traffic in 2011 according to the report by Cicso [14]). *This motivates us to study the QoS satisfaction game in this paper.* Rather than assuming that users wish to increase their data rates whenever possible, we assume that each user has a fixed QoS requirement. If the requirement is satisfied, then the user has no inclination to change his choice of resource. The concept of focusing upon satisfaction rather than data rate maximization was inspired by [15], [16].

We propose a new framework of *QoS satisfaction games*, where satisfaction of a user's QoS requirements depends on its congestion level (i.e., how many users competing with the user for the same spectrum band). The central theme of our results is that allowing users to selfishly and distributively share the spectrum has many advantages over the centralized

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optimization approach. First of all, we show that selfish spectrum sharing is feasible (with Theorem 1), by proving that selfish players can quickly organize themselves into pure Nash equilibria. Second, we show that an efficient centralized optimization is often unfeasible (with Theorem 2), by proving that maximizing social welfare is an NP hard problem. Third, we show that in many cases the game based solution is close to optimal, by bounding the price of anarchy of our systems (with Theorem 3). Together these three results suggest that a decentralized approach towards QoS satisfaction will often be highly effective. We also consider the case where the users are homogenous, and show that the social optima are precisely the same as the pure Nash equilibria in this case (with Theorem 4).

II. QoS Satisfaction Game

A. Game model

A **QoS satisfaction game** is defined by a triple $(\mathcal{N}, \mathcal{C}, (Q_n^c)_{n\in\mathcal{N},c\in\mathcal{C}}, (D_n)_{n\in\mathcal{N}})$ where:

- $\mathcal{N} \triangleq \{1, .., N\}$ is the set of **wireless users**, also referred as the **players**.
- $\mathcal{C} \triangleq \{1, .., C\}$ is the set of **real channels**. Each user can access at most one real channel at a time. Furthermore, we use 0 to represent the **virtual channel**. This will be useful when a user's QoS requirement cannot be satisfied due to limited resource, then the user can choose to cease its transmission to save power consumption (i.e., choose the virtual channel). In summary, each user/player has a strategy set $\tilde{\mathcal{C}} \triangleq \{0, 1, .., C\}$. The **strategy profile** of the game is given as $\boldsymbol{x} = (x_1, x_2, .., x_N) \in \tilde{\mathcal{C}}^N$, where each user n chooses a (virtual or real) channel $x_n \in \tilde{\mathcal{C}}$.
- $Q_n^c(\cdot)$ is a non-increasing function that characterizes user n's data rate in terms of the **congestion level** $I^c(\boldsymbol{x})$. The congestion level $I^c(\boldsymbol{x}) = |\{n \in \mathcal{N} : x_n = c\}|$ in strategy profile $\boldsymbol{x}$ is the number of user who choose the real channel c. Like in many studies of congestion games for spectrum sharing (e.g., [6], [9], [17]), we assume that a user's data rate depends on the number of contending users. This is true when the network adopts a medium access mechanism such as TMDA or CSMA (with the same user window size). We also allow user specific data rate functions, i.e., different users may have different Q_n^c even on the same channel c. This will allow users to have different transmission technologies, choose different coding/modulation schemes, and experience different channel conditions.
- $D_n \geq 0$ is the data rate required by user n to support its applications. For example, listening to an MP3 online will require a small D_n, whereas watching a high definition streaming video requires a large D_n.

User n's **utility** in strategy profile $\boldsymbol{x}$ is

$$U_n(\boldsymbol{x}) = \begin{cases} 1, & \text{if } x_n \neq 0 \text{ and } Q_n^{x_n}(I^{x_n}(\boldsymbol{x})) \geq D_n, \\ 0, & \text{if } x_n = 0, \\ -1, & \text{if } x_n \neq 0 \text{ and } Q_n^{x_n}(I^{x_n}(\boldsymbol{x})) < D_n. \end{cases} \quad (1)$$

A **satisfied user** is a user n with a utility equal to 1, i.e., its received data rate $Q_n^{x_n}(I^{x_n}(\boldsymbol{x}))$ is above or equal to its QoS requirement D_n. A **dormant user** is a user n choosing the virtual channel $x_n = 0$. Such a dormant user gains no benefit for successful channel usage and receives no penalty for expending power, and so it gets a utility of $U_n(\boldsymbol{x}) = 0$. A **suffering user** is a user n with a utility of -1, i.e., its received data rate $Q_n^{x_n}(I^{x_n}(\boldsymbol{x}))$ is lower than its QoS requirement D_n. Such a suffering user expends power without gaining benefit, and so it gets a utility of $U_n(\boldsymbol{x}) = -1$.

It is worth noting that we can easily generalize our model by allowing a user n to receive a utility of u_n if it is satisfied, v_n if it is dormant, and t_n if it is suffering, where $u_n > v_n > t_n$. Making this generalization does not effect the better response dynamics or the set of pure Nash equilibria discussed later on, because the preference orderings of the strategies in the generalized game are the same as in our current model[1]. Our results about convergence (Theorem 1) and computational complexity (Theorem 2) also remain true for games with generalized utility functions. However, since the generalized games allow different users to receive different utilities when satisfied, our result about price of anarchy (Theorems 3) may not hold with the generalized games. In this paper, we will restrict our attention to the utility choices of 1, 0 and -1 for simplicity[2]. The case of generalized utility functions will be further explored in a future work.

B. Key game concepts

*Definition 1 (**Social Welfare**):* The social welfare of a strategy profile $\boldsymbol{x}$ is the sum of all users' utilities, i.e., $\sum_{n=1}^N U_n(\boldsymbol{x})$.

*Definition 2 (**Social Optimum**):* A strategy profile $\boldsymbol{x}$ is a social optimum when it maximizes social welfare.

*Definition 3 (**Better Response Update**):* The event where a user n changes its choice of channel from x_n to c is a better response update if and only if $U_n(c, x_{-n}) > U_n(x_n, x_{-n})$ where $x_{-n} = (x_1, .., x_{n-1}, x_{n+1}, .., x_N)$ is the strategy profile of all users except n.

*Definition 4 (**Pure Nash Equilibrium**):* A strategy profile $\boldsymbol{x}$ is a pure Nash equilibrium if no users at $\boldsymbol{x}$ can perform a better response update, i.e., $U_n(x_n, x_{-n}) \geq U_n(c, x_{-n})$ for any $c \in \tilde{\mathcal{C}}$ and $n \in \mathcal{N}$.

*Definition 5 (**Finite Improvement Property**):* A game has the finite improvement property if *any* asynchronous better response update process (i.e., no more than one user updates the strategy at any given time) terminates at a pure Nash equilibrium within a finite number of updates.

C. Interference threshold form transformation

To ease exposition, we will introduce an equivalent interference threshold form for the QoS satisfaction game. The

[1] Technically speaking, our game is weakly isomorphic [18] to this generalized version.

[2] As shown later, under such a utility setting, the number of satisfied users at both social optima and Nash equilibria equals to the sum of all users' utilities at these equilibria.

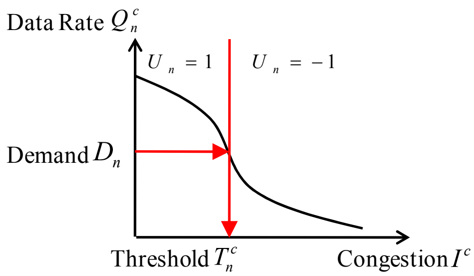

Fig. 1. Illustration of the interference threshold from transformation.

key idea is to consider the critical congestion threshold below which a user's QoS demands can be met. Since the data rate function $Q_n^c(I^c)$ is non-increasing with the congestion level I^c, there must exist a critical threshold value T_n^c, such that $Q_n^c(I^c) \geq D_n$ if and only if the congestion level $I^c \leq T_n^c$ (see Figure 1 for an illustration). Formally, given a pair of (Q_n^c, D_n), we can construct the threshold T_n^c so that

- if $Q_n^c(I^c) < D_n$ for each $I^c \in \{1,..,N\}$ then $T_n^c = 0$,
- if $Q_n^c(I^c) > D_n$ for each $I^c \in \{1,..,N\}$ then $T_n^c = N+1$,
- otherwise T_n^c is equal to the maximum integer $I^c \in \{1,..,N\}$ such that $Q_n^c(I^c) \geq D_n$.

These conditions guarantee that

$$Q_n^c(I^c) \geq D_n \Leftrightarrow I^c \leq T_n^c, \tag{2}$$

for each real channel c. We can then express a QoS satisfaction game $g = (\mathcal{N}, \mathcal{C}, (Q_n^c)_{n\in\mathcal{N},c\in\mathcal{C}}, (D_n)_{n\in\mathcal{N}})$ in the *interference threshold form* $g' = (\mathcal{N}, \tilde{\mathcal{C}}, (T_n^c)_{n\in\mathcal{N},c\in\mathcal{C}})$. The utility of user n can be computed accordingly as

$$U(\boldsymbol{x}) = \begin{cases} 1, & \text{if } x_n \neq 0 \text{ and } I^{x_n}(\boldsymbol{x}) \leq T_n^{x_n}, \\ 0, & \text{if } x_n = 0, \\ -1, & \text{if } x_n \neq 0 \text{ and } I^{x_n}(\boldsymbol{x}) > T_n^{x_n}. \end{cases} \tag{3}$$

The interference threshold form transformation reduces the size of parameters by replacing (Q_n^c, D_n) with T_n^c. Moreover, the result in (2) ensures that the original game g is equivalent to the game g', since the utility $U_n(\boldsymbol{x})$ received by user n in g is the same as that received by user n in g' for every strategy profile $\boldsymbol{x}$ and user n. For the rest of the paper, we will analyze the QoS satisfaction game in the interference threshold form. **Due to lack of space, we have included all detailed proofs in the online technical report [19].**

III. Properties of QoS Satisfaction Game

Now we explore the properties of QoS satisfaction games, including the existence of pure Nash equilibrium and the finite improvement property.

A. Characterization of pure Nash equilibria

First of all, it is easy to see that at a pure Nash equilibrium each user must be either satisfied or dormant. To see this, consider a strategy profile $\boldsymbol{x}$ where a user n is suffering (i.e., its utility is -1). Then user n can do a better response update by changing its channel to 0 and becoming dormant. The presence of a player that can do a better response update implies that $\boldsymbol{x}$ is not a pure Nash equilibrium, in this case. Similarly, there are also no suffering users at a social optimum, because having a suffering user becoming dormant increases their utility without decreasing any other user's utility. Next we show in Theorem 1 that every QoS satisfaction game has the finite improvement property. This guarantees the existence of a pure Nash equilibrium.

Theorem 1: Every N-user QoS satisfaction game has the finite improvement property. Any asynchronous better response updating process is guaranteed to reach a pure Nash equilibrium, within no more than $2N + 3N^2$ updates.

Proof: We define a function $\Phi(\boldsymbol{x}) = \sum_{n\in\mathcal{N}} F_n(\boldsymbol{x})$ which maps each strategy profile $\boldsymbol{x}$ to an integer. Here we have

$$F_n(\boldsymbol{x}) = \begin{cases} 2T_n^{x_n} - I^{x_n}(\boldsymbol{x}), & \text{if } x_n \neq 0 \\ 0, & \text{if } x_n = 0, \end{cases} \tag{4}$$

for each player $n \in \mathcal{N}$.

Next we show that the value of the function Φ increases by at least one with each better response update. Consider the generic case where the system is in a strategy profile $\boldsymbol{x}$ and then some player $n' \in \mathcal{N}$ does a better response update, and changes his channel from $x_{n'} = c' \in \tilde{\mathcal{C}}$ to $d' \in \tilde{\mathcal{C}}$. Let $\boldsymbol{y} = (x_1, .., x_{n'-1}, d', x_{n'+1}, .., x_N)$ denote the new strategy profile which results from this better response update. Now we can show that $\Phi(\boldsymbol{y}) \geq \Phi(\boldsymbol{x}) + 1$ for each of the three possible *types* of better response updates that player n' can make (depending upon whether c' and d' represent real or virtual channels).

The first case we consider is where the active player n' changing from a real channel $c' \neq 0$ to the virtual channel $d' = 0$ (the user stops using a bad channel). The change in Φ caused by this update is

$$\Phi(\boldsymbol{y}) - \Phi(\boldsymbol{x}) = F_{n'}(\boldsymbol{y}) - F_{n'}(\boldsymbol{x}) + \sum_{n\in\mathcal{N}:n\neq n'} F_n(\boldsymbol{y}) - F_n(\boldsymbol{x}). \tag{5}$$

Since $y_{n'} = d' = 0$ is the virtual channel, we have

$$F_{n'}(\boldsymbol{y}) = 0. \tag{6}$$

Also, since $x_{n'} = c' \neq 0$ is a real channel, we have

$$F_{n'}(\boldsymbol{x}) = 2T_n^{c'} - I^{c'}(\boldsymbol{x}). \tag{7}$$

On channel c', there are $I^{c'}(\boldsymbol{x}) - 1$ users sharing the channel with user n' in the strategy profile. Each of these users n has $F_n(\boldsymbol{y}) = F_n(\boldsymbol{x}) + 1$, as the congestion level on this channel decreases. For another player n that does not use channel c' at $\boldsymbol{x}$, we have $F_n(y) = F_n(x)$. It follows that

$$\sum_{n\in\mathcal{N}:n\neq n'} F_n(\boldsymbol{y}) - F_n(\boldsymbol{x}) = I^{c'}(\boldsymbol{x}) - 1. \tag{8}$$

Substituting Equations (6),(7) and (8) into Equation (5) gives

$$\Phi(\boldsymbol{y}) - \Phi(\boldsymbol{x}) = 2I^{c'}(\boldsymbol{x}) - 2T_n^{c'} - I^{c'} - 1. \tag{9}$$

Since it is a better response update for n' to stop using c' we have $U_{n'}(\boldsymbol{x}) = -1$, and so $I^{c'}(\boldsymbol{x}) \geq T_n^{d'} + 1$. Combining this with Equation (9) yield $\Phi(\boldsymbol{y}) \geq \Phi(\boldsymbol{x}) + 1$, as required.

One must also consider the cases where $c' = 0$ and $d' \neq 0$ (i.e., the active player starts using a new real channel), and the case where $c' \neq 0$ and $d' \neq 0$. For each of these two cases we can show that $\Phi(\boldsymbol{y}) \geq \Phi(\boldsymbol{x}) + 1$ (with details are given in the online technical report [19]). In this way one proves that Φ increases by at least one under each type of better response update.

Finally we show that Φ is bounded above and below and hence the asynchronous better response will stop within a finite number of steps. Note that for any strategy profile $\boldsymbol{z}$ and any player n, we have $-N \leq -I^{z_n}(\boldsymbol{z}) \leq F_n(\boldsymbol{z}) \leq 2T_n^{z_n} \leq 2N + 2$. It follows that $-N^2 \leq \Phi(\boldsymbol{z}) \leq 2N^2 + 2N$. Combined with the fact that each better response will increase Φ by at least one, this implies that it shall take no more than $2N + 3N^2$ better responses to increase the Φ value from the minimum to the maximum. When the initial strategy profile corresponds to a Φ value larger than $-N^2$, it will take less than $2N+3N^2$ steps to stop. This implies that when we evolve the system under better response updates, we must reach a strategy profile $\boldsymbol{w}$ from which no further better response updates can be performed, within $2N + 3N^2$ time slots. Such a strategy profile $\boldsymbol{w}$ must be a pure Nash equilibrium by definition. ■

Theorem 1 implies that the general QoS satisfaction games (with heterogenous channels and users) can self organize into a stable state within polynomial time.

B. Finding a social optimum is NP hard

Although Theorem 1 implies that pure Nash equilibria are easy to achieve, it turns out that finding a social optimum can be extremely challenging (as our next result implies).

Theorem 2: The problem of finding a social optimum of a QoS satisfaction game is NP hard.

Notice that a social optimum in general may not be a pure Nash equilibrium. To prove Theorem 2, we can show that the 3-dimensional matching decision problem (which is well known to be NP complete [20]) can be reduced to the problem of finding a social optimum of a QoS satisfaction game where thresholds $T_n^c \in \{1, 3\}$ for each n and c. See the technical report [19] for a complete proof. Theorem 2 provides the major motivation for our game theoretic study, because it asserts that the centralized network performance optimization is fundamentally difficult. It therefore makes sense to explore decentralized alternatives such as game based spectrum allocation.

C. Price of anarchy

Although Theorem 2 implies that finding a social optimal strategy profile can be fundamentally difficult, we do know from Theorem 1 that pure Nash equilibria can be found with relative ease. This naturally raises the question of how the social welfare of pure Nash equilibria compare with that of social optima. In other words, *how much social welfare will be lost by allowing the users to organize themselves, rather than directing them to a social optimum?*

To gain insight into this issue, we study the price of anarchy (PoA) [21]. Recall that $\tilde{\mathcal{C}}^N$ is the set of strategy profiles of our game. Let $\Xi \subseteq \tilde{\mathcal{C}}^N$ denote the set of pure Nash equilibria of our game. Note that Theorem 1 implies that Ξ is non-empty.

Now the **price of anarchy**

$$\mathrm{PoA} = \frac{\max\{\sum_{n=1}^N U_n(\boldsymbol{x}) : x \in \tilde{\mathcal{C}}^N\}}{\min\{\sum_{n=1}^N U_n(\boldsymbol{x}) : x \in \Xi\}},$$

is defined to be the maximum social welfare of a strategy profile, divided by the minimum social welfare of a pure Nash equilibrium[3]. The social welfare of a system at a pure Nash equilibrium can be increased by at most PoA times by switching to a centralized solution.

Theorem 3: Suppose we have a QoS satisfaction game $(N, C, (T_n^c)_{n \in \mathcal{N}, c \in \mathcal{C}})$ where $T_n^c > 0$[4] for each user n and each real channel c. The price of anarchy of this game satisfies

$$\mathrm{PoA} \leq \min\left\{N, \frac{\max\{T_n^c : n \in \mathcal{N}, c \in \mathcal{C}\}}{\min\{T_n^c : n \in \mathcal{N}, c \in \mathcal{C}\}}\right\}.$$

The central idea behind the proof is as follows If the minimum social welfare of a pure Nash equilibrium is less than N, then we must have that (i) the maximum social welfare of a strategy profile is no greater than C times the maximum threshold T_n^c, and (ii) the minimum social welfare of a pure Nash equilibrium is no less than C times the minimum threshold T_n^c. See the technical report [19] for a complete proof. Theorem 3 implies that the performance of every pure Nash equilibrium will be close to optimal when the minimum threshold of a user-channel pair is close to the maximum threshold of a user-channel pair. This is a quite nice result, when one considers that pure Nash equilibria can be quickly reached by better response updates (Theorem 1) while finding social optima is NP hard (Theorem 2).

D. QoS satisfaction games with homogenous users

Motivated by Theorem 3, we examine the special case of QoS satisfaction games with homogenous users. In this case social optima which are pure Nash equilibria can easily be found. We say that a QoS satisfaction game has **homogenous users** when $T_1^c = T_2^c = \ldots = T_N^c$, for each $c \in \mathcal{C}$ (i.e., each user has the same threshold for any real channel c). This corresponds to the case that all users have the same data rate function Q_n^c on the same channel c (but they may have different data rates on different channels) and the same QoS requirement D_n. For example, spectrum sharing in a network of RFID tags on a warehouse may correspond to such a QoS satisfaction game, because every device experiences the same environment and requires a similar data rate to operate.

When discussing QoS satisfaction games with homogenous users, we drop the subscripts and use T^c to denote the common threshold for all users on channel c. Since users are homogenous, we only need to keep track of how many users choose each channel in order to describe the game

[3] In our case the PoA equals the maximum number of users that can be satisfied, divided by the minimum number of users that can be satisfied at a pure Nash equilibrium.

[4] This constraint insures that some user will be satisfied in every pure Nash equilibrium of the game, and avoids the possibility of the PoA involving 'division by zero'.

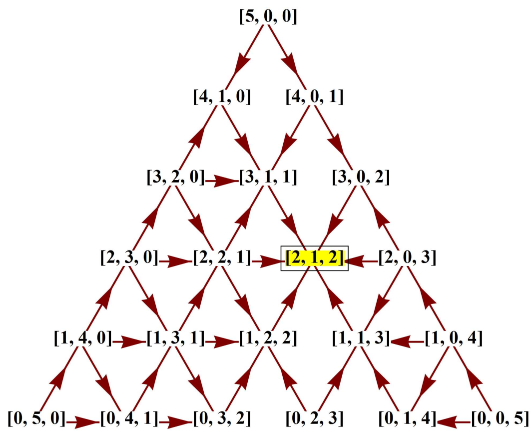

Fig. 2. An illustration of the better response dynamics of the game with 5 homogenous users, and two real channels with thresholds $T^1 = 1$ and $T^2 = 2$. The point $[I^0, I^1, I^2]$ represents the scenarios where there are I^0, I^1, and I^2 users on channels 0, 1, and 2 respectively. Every arrow represents a potential state transition that can occur due to a best response update. Note that we only keep track of the number of users of each channel, and that individual points may correspond to multiple strategy profiles. For example, the point $[I^0, I^1, I^2] = [1, 4, 0]$ corresponds to each strategy profile in the set $\{(0, 1, 1, 1, 1), (1, 0, 1, 1, 1), (1, 1, 0, 1, 1), (1, 1, 1, 0, 1), (1, 1, 1, 1, 0)\}$. The unique point corresponding to the pure Nash equilibria is shaded yellow.

dynamics (see Figure 2). Next we will show that any pure Nash equilibrium in QoS satisfaction games with homogenous users is also a social optimum.

Theorem 4: Let $\boldsymbol{x}$ be a strategy profile of a QoS satisfaction game with N homogenous users and C real channels, with thresholds $T^1, T^2, \ldots, T^C$. The following three statements are equivalent:

1) $\boldsymbol{x}$ is a pure Nash equilibrium;
2) $\boldsymbol{x}$ is a social optimum;
3) There are no suffering users in $\boldsymbol{x}$ and the number of satisfied users is $\min\{N, \sum_{c=1}^{C} T^c\}$.

Theorem 3 implies that each pure Nash equilibrium is a social optimal in the special case where resources and users are homogenous. Theorem 4 is more than just a corollary of Theorem 3, since it states that this remains true when only the *users* are homogenous, and also states that the converse holds. Theorems 1 and 4 together imply that asynchronous better response updating will always converge to a social optimum in polynomial time when the game has homogenous users. Moreover, Theorem 4 implies that when $\sum_{c=1}^{C} T^c \geq N$, there exists a satisfaction equilibrium (as defined in [15]) where all the users can be satisfied.

IV. Conclusion

In this paper, we proposed a framework of QoS satisfaction games to model the distributed QoS satisfaction problem among wireless users. The game based solution is motivated by the observation that computing the global optimal solution is an NP hard problem. We have explored several aspects of QoS satisfaction games including the convergence dynamics and the price of anarchy, and our results reveal that selfish spectrum sharing can be a very effective way to allow users to meet their QoS requirements. In the future, we will also look at the case where the channels are homogenous, and our preliminary results suggest that it is possible to design fast algorithms to find social optima. We shall also consider *QoS satisfaction games on graphs* -which take account of spatial reuse by using a graph to represent which users are close enough to interfere with one another. Also, we shall examine the scenario where users can access multiple channels simultaneously.

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